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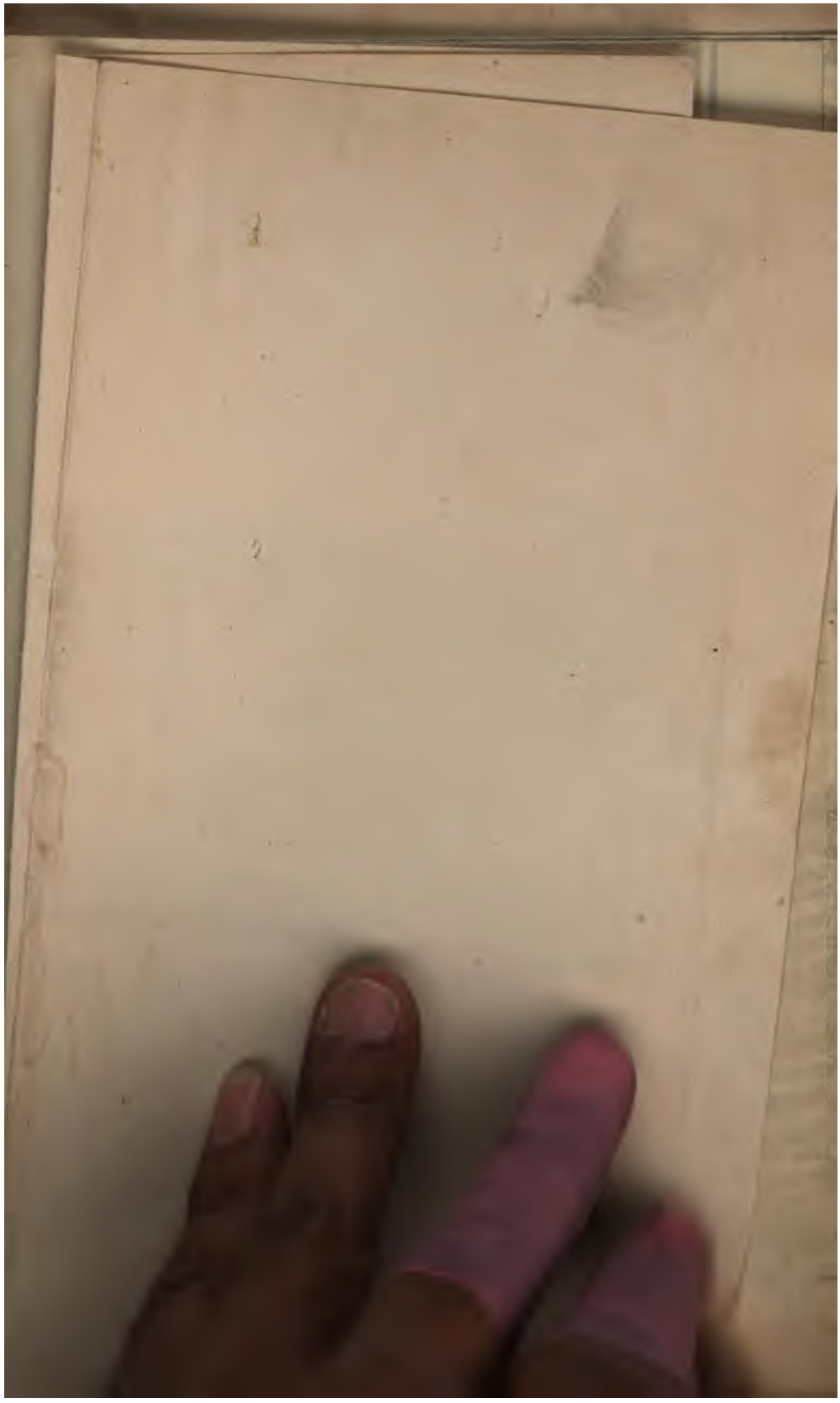

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Road Connections
by the
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Scale : One Inch to 4200 Feet
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[redacted]

REPORT

OF THE

ENGINEER-IN-CHIEF

OF THE

ILLINOIS AND ST. LOUIS

BRIDGE COMPANY.

ST. LOUIS, MAY, 1868.

ST. LOUIS:
MISSOURI DEMOCRAT BOOK AND JOB PRINTING HOUSE, CORNER FOURTH AND PINE STREETS.
1868.

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REPORT.

To the President and Directors of the Illinois and St. Louis Bridge Company:

GENTLEMEN—I have the honor to submit the following report of the operations of the engineering department of your Company.

In view of the great importance of your enterprise, the deep interest manifested in it by our citizens and the public generally, and because the plans adopted by you have been frequently misrepresented and unfairly criticised, I have deemed it proper that everything of interest connected with my department should be placed in such form as to be clearly understood, not alone by your stockholders, but also by every person of ordinary intelligence in the community. I have, therefore, endeavored to explain the plan of structure, the principles involved in its construction, and the reasons for its preference, in the simplest language I can command, and with an avoidance, as far as possible, of the use of all technicalities not understood by every one. At the same time, you will be furnished, in an appendix, with all the scientific data, principles and formulæ involved in investigating and calculating the various strains to which each and every part of the Bridge is liable to be subjected. These are so arranged, together with the results deduced therefrom, as to furnish to the most critical and scientific engineer, the materials in convenient form whereby he may be able with great economy of labor, to investigate the correctness of every step that I propose to take in constructing your Bridge.

Fully estimating the great responsibility assumed in undertaking to design and complete this important work, I have felt that in no way could I so certainly insure a successful result, and at the same time manifest my appreciation of the obligation imposed upon me, as by securing the aid of the ablest talent, in every department of the work, and proving by careful experiment as far as possible, everything connected with it that has not been already fully demonstrated in practice, so that when the whole is finally consummated,

you can feel assured no step was taken that was not well considered with due regard to the safety, durability, and economy of the structure.

The mathematical investigations and calculations for the Bridge were confided to my chief assistant, Col. Henry Flad, C. E., and in this laborious duty he has been faithfully and efficiently aided by Mr. Chas. Pfeifer, C E. Several months of patient labor have been spent by these gentlemen in the investigation of the arch with spandrel bracings, the ribbed arch with pivoted ends (as in the Coblentz bridge), and with fixed ends, and of various depths. After careful revision by Col. Flad, the results obtained from time to time were submitted to me; and, finally, to guard against any possible error in the application of the principles upon which the investigations were made, or in the results arrived at, they were referred by me to the patient analysis and careful examination of Chancellor W. Chauvenet, LL. D., of the Washington University, formerly Professor of Mathematics in the U. S. Naval Academy at Annapolis. His certificate, affirming their correctness in every particular, will be found appended to this report. For the interest this gentleman has taken in the enterprise, for the care bestowed in examining and verifying the scientific data required for the work, and for many valuable suggestions and simplifications in the investigation, I feel under many obligations.

It gives me great pleasure also to state in this connection that Chancellor Chauvenet, whose eminent ability as a mathematician is known and acknowledged throughout Europe as well as America, accords high praise to Col. Flad, and also to Mr. Pfeifer, for the correctness and ability shown by them in discharging the important duty confided to them.

LOCATION.

As the cost of the Bridge, its revenues, and the public convenience are greatly affected by the location of the structure, I have given this subject the utmost consideration. To aid me in the solution of this important question, I called to my assistance, with your permission, Mr. R. B. Lewis, C. E., a gentleman of great experience and high reputation as a locating engineer, and for many years in the service of the Pennsylvania Railroad. He is now President of the Broad Top Railroad and Consulting Engineer of the St.

Louis and Vandalia Railroad. For his valuable aid and advice I feel greatly indebted. His opinion, and that of my entire corps of assistants, after a thorough examination of the subject, was unqualifiedly in favor of the location at Washington avenue, which location was formally adopted by the Board of the St. Louis and Illinois Bridge Company, and, since the consolidation of the two companies, by the Board of the consolidated Company also.

When considered with reference to the cost of the structure, the location at Washington avenue commends itself, because at this point the river is narrower than at any other place opposite the city. It is nearly two hundred feet less than at the location selected for the bridge designed to be built by Mr. Boomer, and is 580 feet less than at Venice (opposite the northern part of the city), where the location has been urged as more suitable for railroad purposes. As the Bridge proper must at either of these two places be longer than at Washington avenue, and as there can be no saving in the cost of its approaches or foundations at either locality over the one selected for it, either one of those locations if selected, would result in an inevitable increase in the cost of the Bridge. I am not unmindful of the fact that the cost of the tunnel, in connection with the location at Washington avenue, is advanced as an objection to this selection.

The location selected for Mr. Boomer's bridge, a few blocks above Washington avenue, contemplated the running of steam trains across Broadway and other great thoroughfares and out through Cass avenue. The tunnel is no more a necessity at Washington avenue than at the latter location. A grade of thirty feet to the mile would bring the trains from the centre of the centre span on your Bridge to the level of Washington avenue at Third street; and our citizens would be just as likely to permit them to traverse that avenue above ground as they would the route designed for them at the other location. It is preposterous to believe that the city would permit steam trains to pass over either route above ground, both being through portions of the city densely populated, and both crossing the same great thoroughfares. At either location a tunnel is a necessity for unobstructed railway traffic. A tunnel in connection with the Boomer location would necessarily be longer than at Washington avenue, and would therefore cost more. As the cost of your tunnel can not exceed $400,000, and as the Company will be entitled to charge

tolls on trains passing through it, that part of your investment must prove profitable, and is therefore unobjectionable.

Washington avenue is the centre of population of the city of St. Louis. It is a spacious and elegant avenue, dividing the city into two nearly equal portions of territory. The city front on the river extends three and three-quarter miles below the foot of this avenue and three and one-quarter miles above it, thus nearly equally dividing the seven miles of wharf or river front that forms the eastern boundary of your city. The grades upon this avenue are of the gentlest character, from its western end to Third street. From this point the carriage-way of the Bridge will form a continuation of the avenue to Third street in East St. Louis. In the latter city the eastern approach to the Bridge will commence at the intersection of Crooks and Third streets. Vehicles starting in East St. Louis on Crooks street at Fourth, to cross the Bridge, will reach the top of the eastern abutment by ascending a grade of but five feet rise in one hundred. After crossing the Bridge to the western abutment, the grade of the carriage-way is perfectly level to the intersection of Third street in St. Louis. Here the wide and level avenue will form a most convenient outlet for the vast tide of travel constantly crossing the Mississippi. The intersection of Washington avenue by streets at right angles to it, every three hundred feet, will greatly facilitate the exit and entrance of teams and passengers from and on to the Bridge. Two of these streets (Fourth and Fifth) are each 80 feet wide.

The carriage-way of the Bridge will be of sufficient width (34 feet) to accommodate two vehicles going abreast in each direction, and will have a double track for horse railways laid through its centre. On each side of the carriage-way will be a foot-path eight feet wide for pedestrians. Convenient stairways will be provided on the wharf on each side of the river, and at Main and Second streets, for entrance to and exit from the Bridge.

When we consider that one-half of the people of St. Louis reside north, and the remainder south, of Washington avenue; that it divides the territory of the city into two nearly equal portions, and that teams and passengers can enter and leave the Bridge with such ease at either end of the structure, I think no one will deny that this location is an excellent one so far as its revenues can be affected by local travel and traffic.

The depot or depots for freight to be received at or shipped from St. Louis must, because of the greater cheapness of the ground required, be located on the opposite shore. Nowhere on this side, within a reasonable distance of the business centre of your city, could the requisite quantity of land for a great union freight depot be obtained except at immense cost. Opposite the city, and near the eastern terminus of the Bridge, however, this need can be supplied at a comparatively moderate price. The Bridge and its approaches being less than three-quarters of a mile long, and having its western terminus in the very heart of the business quarter of St. Louis, it follows that by this location, and the establishment of freight depots near the eastern terminus of the Bridge in Illinois, goods can be carted within more moderate and convenient distances from or to the depots on the other side and the warehouses in St. Louis, than would be possible by any other location.

When the location is considered with reference to expediting and cheapening the transportation of through freights and passengers arriving and departing by any of the ten railroads that are now to be provided with such facilities by this Bridge, it becomes apparent that a location that is not central can only benefit some few of them, at the expense of the others. The roads which are now constructed from the East all converge to a space not one mile in extent, on the opposite shore, and their termini are all nearly opposite the centre of the city. From the western end of the Bridge, at Washington avenue and Third street, a tunnel 5,000 feet long will extend under Washington avenue and Eleventh street, connecting the railway tracks on the Bridge with the low grounds forming the bed of the old Chouteau pond, on which the Pacific track is laid. The North Missouri, the Southwest Pacific, and the Iron Mountain roads, can all be brought through the valley by which the Pacific road reaches the site of the Chouteau pond, at a moderate cost; and the erection of a union passenger depot near the end of the tunnel will enable all four of the Missouri roads to receive from, and exchange passengers with the roads on the Illinois shore, whose trains can all be run into this depot, where they will receive and deposit their passengers and baggage. Through the tunnel and the Bridge the freight trains from Missouri can be taken to the union freight depot in East St. Louis, and there receive, discharge, or exchange their cargoes. Of course,

the establishment of these two union depots for freight and passengers will not prevent the several roads from having such other depots as they may find useful. The union depots will be required chiefly to facilitate the rapid and cheap transfer of freights, passengers, and baggage from one road to another.

I am aware that many persons maintain that facilitating the transfer of freights and passengers will cause our people to lose the profit that arises from hauling them from depot to depot, and in entertaining the passengers at our hotels and boarding houses. This policy is so short-sighted as scarcely to need noticing. The great struggle now being made by so many rival lines and cities for the trade of the West, should admonish us that unless we too offer inducements on the score of economy, convenience, and dispatch, we can scarcely hope to maintain what we now have. Let us secure this trade by expediting it in every way in our power, and by lessening the burdens with which it is now taxed. The marvelous rapidity with which it will then multiply on our hands will give our people greatly increased profit and employment.

In deference to the views of railroad gentlemen of great experience, I caused surveys and examinations to be made to determine the propriety of locating the Bridge at the northern part of the city, opposite the town of Venice.

It was asserted that at this locality the bed rock of the river formed the bottom of the channel, and that the piers could be cheaply and readily erected on it. Borings made by me near the Illinois shore, to ascertain the truth of this assertion, prove the rock to be overlaid with 60 feet of sand. On the Missouri side, examinations made by the Board of Water Commissioners show the rock to be 30 feet below the sand. Borings made by the late Mr. Homer, City Engineer, by Colonel Bissell, C. E., and by myself, at various places between the one in question and Washington avenue, leave no reason to doubt that the rock there slopes with the same regularity from west to east that we know it does at other localities in this harbor, and is nearly or quite as far below the surface as at Washington avenue.

The river at that point being much wider than at Washington avenue, the erection of a bridge there would involve the necessity of more piers, and a greater length of superstructure. The extensive plateau on this side, scarcely above the city directrix,

would require the construction of an expensive approach of from 1,500 to 2,000 feet, according to the grade, while it would not be possible to connect with any Missouri road, except the North Missouri, without the construction of several miles of road, extending through the public streets of North St. Louis, and thence by a circuitous and expensive route back of the city to the Pacific road. From Rocky Branch, in the extreme northern part of St. Louis, to the valley half a mile south of Washington avenue, through which the Pacific road comes in, there extends an almost unbroken ridge or plateau, varying from 75 to 125 feet above the level of the Pacific track at Eleventh street. This ridge would interpose expensive difficulties in connecting the structure with any Missouri road except the North Missouri. In Illinois, only the two roads coming in from the northeast could use the Bridge advantageously when connecting with the North Missouri. A bridge thus located could only be used by the Missouri Pacific, the Southwest Pacific, and Iron Mountain, on this side, by the construction of an expensive piece of road of three or four miles in length; and in connecting those roads with the Ohio and Mississippi, the Vandalia road, the Belleville, and the Cairo roads, on the other side, they would do so at a loss of from three to five miles in distance. Of course, this location is only advocated for a railroad bridge. Local traffic on it, except to a very limited amount, could not be expected.

For railroad purposes, except in the interest of the North Missouri, Alton and Terre Haute, and St. Louis and Chicago railways, the location has nothing whatever to recommend it, while the advantages accruing to those roads from a connection at Venice would not compare with those that will be derived by them from the central location at Washington avenue. By the Venice location, it would be idle to hope for a union depot almost in the very heart of the city, for the convenient transfer of passengers and baggage. By the location at Washington avenue, this is attained without extra cost to the roads in making the connections with each other, and without danger of interruption from over-crowded streets, or risk of injury to persons or property.

As a matter of convenience to the marine interests, the location at Washington avenue must be deemed judicious. It is idle to talk of bridging the river, and planting piers in its channel, without obstructing navigation. No matter how wide the spans may be,

every pier that is placed in the river is an obstruction, calculated to create danger, and cause anxiety to those who navigate it. By the location at Washington avenue, the wharf is nearly equally divided above and below the Bridge. This will make it unnecessary for the steamers trading on the upper rivers to pass under the structure, whilst those engaged on the Ohio and the lower rivers will seldom be required to pass above it. If the Bridge were located in the upper portion of the city, all of the upper river boats would have to pass and repass it every trip.

From all these facts, I feel confirmed in asserting that at no other location could the Bridge be erected so cheaply, at no other one would its revenues be so great, and at no other point opposite your city would the public at large be so well accommodated.

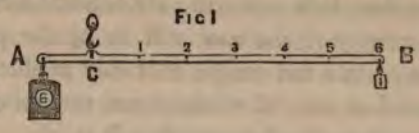
ARCH AND TRUSS BRIDGES.

Because of the frequent assertion that your structure will be needlessly extravagant, I deem it proper to illustrate, in as simple a manner as I possibly can, enough of the general principles involved in the construction of bridges to enable any one to satisfy himself that the plan adopted for the construction of this Bridge, instead of being needlessly expensive, is really the most economical of all known methods. The general principles involved in the construction of an *arch* or a *truss* are not so intricate or difficult but what any one with ordinary intelligence can, with a little explanation, comprehend them sufficiently to judge for himself of the truth of this assertion. I shall do this before proceeding to an explanation of the plan of your proposed Bridge, as the method adopted in it will then be more readily understood and its merits appreciated.

Any one who can be made to understand the principles of the simplest of all the mechanical powers, the *lever*, can readily comprehend the explanation I propose making, and though he may never have reflected upon the subject, a few minutes spent in carefully considering the following illustrations will be sufficient to enable him to understand the economy of the arch, over the truss, for long span bridges.

Suppose the lever A B (Fig. 1) to rest on the fulcrum C, so that the long arm is six times the length of the short one, then it is evident that one ton placed at B will balance six tons at A.

Fig 1



If the short arm of the lever be bent down, as in Fig. 2, one ton at B will exert a force equal to six tons at A in the direction of the arrow; and it will create a pulling or tensile strain on the hook at the same time equal to six tons. If two such levers be placed together, as in Fig. 3, with one ton weight on each at B (two tons on the two), then the strain of six tons on the hook in Fig. 2 will be transferred to B, where the two ends of the levers will press against, and mutually support, each other. To retain the short ends of the levers at D and E from separating, it will be necessary to tie them together with a chord, D E, capable of sustaining a strain of six tons, that being the strain at A (Fig. 2) in the direction of the arrow. Anything interposed between the levers at B would be subjected to a crushing or compressive force of six tons.

Fig 2

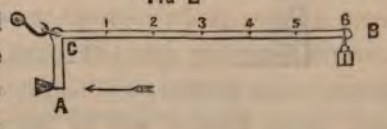
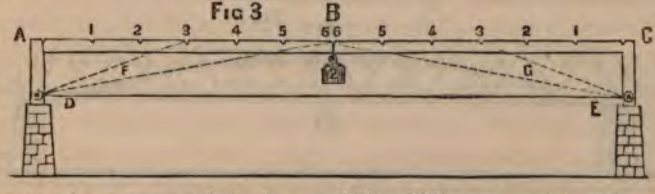


Fig 3



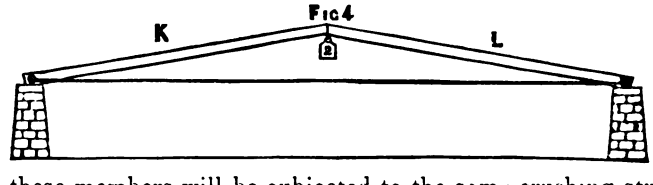
The two long arms of the levers (Fig. 3) here represent the upper or compression member of a truss sustaining a crushing force of six tons, while the chord is the tension member, and is resisting, at the same time, a strain of six tons, that is endeavoring to tear it asunder. If the upper member fails to resist the crushing force, or the lower one is rent asunder, the truss must fall.

To avoid complicating the explanation, the illustration assumes that the levers are perfectly rigid, and makes no account of their weight. If, instead of placing the two tons at B, Fig. 3, we suppose their long arms were each of one ton weight, then the strains would be only half as great; for, if we remove the weight at B,

Fig. 1, and suppose the long arm of the lever to weigh just one ton, then this arm will only balance three tons at A, for the centre of gravity of the arm will be at the point 3 on the lever, and at this point a ton weight will only produce half the strain it would if placed at the end of the lever, the point 3 being only half the distance from the fulcrum that B is.

The strain upon the chord (Fig. 3) comes from the weights at B acting in the direction of the dotted lines, F G; hence, by substituting the straight members, K and L (Fig. 4), for the bent

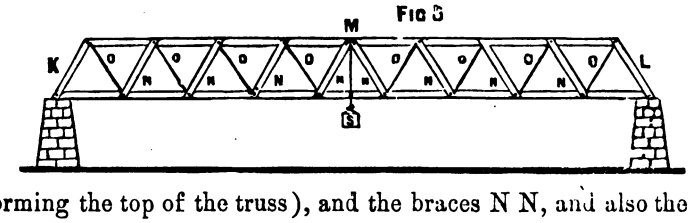
FIG 4



levers, these members will be subjected to the same crushing strain, and, being shorter, will be more economical in material. This simple, triangular truss represents the most economic of all forms of short trusses known.

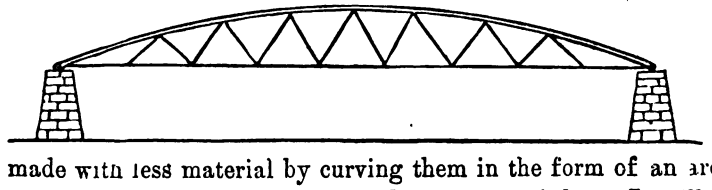
If we desire to extend the span of this truss, the members K and L are liable to bend downwards with their own weight. To obviate this difficulty, many expedients are resorted to; one method is shown in Fig. 5 which is by the introduction of a third member, M

FIG 5



(forming the top of the truss), and the braces N N, and also the tie-rods O O. Here K, M and L, sustain the entire compressive force exerted by the weight of the whole structure. These members may

FIG 6



be made with less material by curving them in the form of an arch, as in Fig. 6. This constitutes the bow-string girder. It will be

observed that as the compressive and tensile strains are about equal, the truss requires about the same quantity of material in the lower member for tension that it does in the upper or compression member, when the material is the same in both, while the latter is really the sole supporting member of the structure.

The illustration of the lever shows that the strain on the compression and tension members is increased by diminishing the height or depth of the truss. For instance: if the short arm of the lever is only one-twelfth of the length of the other arm, instead of one-sixth, as in Fig. 2, the same weight would create twice as much strain at A as before. The truss, in that case, would be twenty-four times its height in length, instead of twelve times, as in the illustration at Fig. 3. Thus, making the height less, requires more material in the upper and lower members, and making it greater, requires more in the braces N N, and in the tension rods O O, as they must then be longer. The proportions that insure the greatest economy are found to be about one-tenth of the length of span for the height, varying however, in different systems from one-eighth to one-twelfth.

The bow-string girder (Fig. 6) requires theoretically probably as little material in its construction, in proportion to the weight to be sustained, as any form of truss known. If we support the compression member or arch of this truss (Fig. 7) between stone abut-

FIG 7

ments strong enough to bear six tons of horizontal thrust, or exactly what its lower member must sustain, we can then dispense with the latter altogether, for it must be evident that its only purpose is to keep the bow or arch from spreading at its ends. If the truss must be supported on piers, it will at once become an interesting question, what will be the difference between the cost of the light piers needed to uphold the truss, and the heavier ones required to sustain the horizontal thrust of the arch. By the horizontal *thrust* of the arch, we mean the strain thrown by it upon the tension member in the truss. Of course, if this member be dispensed with, the abutments

must sustain this thrust; and their ability to sustain it is simply a question of weight and arrangement of stone. The force of the thrust is easily known by calculation, and when we know that one cubic yard of stone will require a certain force to move it, we can readily calculate how many cubic yards of it will be required to resist a given force or thrust. As no account of the bond of the cement is taken in the calculation, we will have that much additional safety in the abutment. If the excess of masonry required for abutments be found to cost less than the tension member, then the arch (or the bow without the chord) will be the cheaper structure. It may occur, too, that because of floods, ice, and drift, it may be prudent to use heavier piers than would otherwise sustain the truss. This would be an additional argument favoring the use of the arch.

By referring to Fig. 6 it will be seen that the bracing between the arch and the chord, as well as almost the whole of the chord itself, is suspended from the arch. In a span of 500 feet, these braces at the centre of the arch would be from 50 to 75 feet long, and their weight and that of the chord would be enormous; yet they bear no part of the load, but serve only to preserve the form of the bow. The sole sustaining member of this truss is therefore the compressive member or arch. It must be evident then, that by sustaining that member between abutments we not only save the cost of the tension member and bracing, but we relieve the arch of this constant and enormous weight also. Now, if this bow-string truss were simply strong enough to bear its own weight before, the same arch supported between abutments, as in Fig. 7, and relieved of this weight, would then sustain an imposed load on the Bridge equally as great as the weight of the tension member and braces taken away. Indeed, if the span of the truss were 500 feet, these needless members would equal the weight of two loaded trains of cars throughout its entire length. It will be asserted that the tension member is all that is saved by using the abutments, because the bracing is needed to preserve the form of the arch also, when but one-half of the span is loaded, whether we use the tension chord or the abutments. This is true, but in the latter case a much smaller quantity of bracing material is needed, as I shall soon prove.

I have purposely taken that form of truss which is of all others the most favorable to those who may be disposed to question the propri-

ety of using the arch in the Bridge at this location, because in no other form of truss with the distributed load, even in theory, can the braces be omitted. (In the tubular girder, the vertical plates forming the sides of the truss constitute the bracing between the upper and lower members.)

Referring to Fig. 7, if this arch be of equal weight throughout its length, and parabolic in form, with a load equally distributed, it will be self-sustaining, and will require no bracing; but when a moving load at A (Fig. 8) has covered that end of the arch,

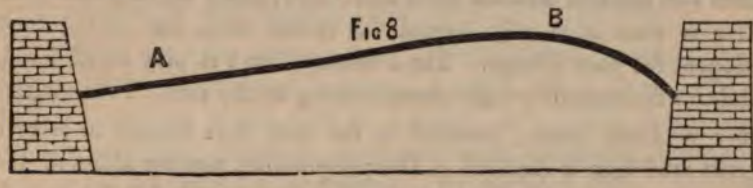


Fig 8

it will be straightened, and the unloaded portion at B will be bent upward.

As the strength of the arch is dependent upon its form, it is necessary to adopt such means as will preserve it in shape under all trials to which it may be subjected. The usual method of counteracting the effect of the moving load, is by spandrel bracing (Fig. 9). Here A B is a member extending over the arch from pier to

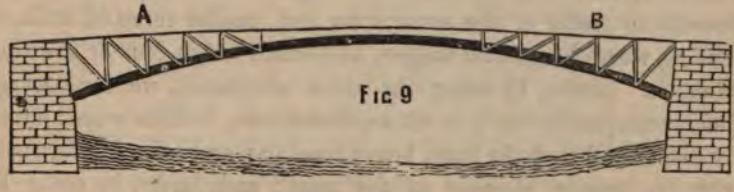


Fig 9

pier. To this member are secured braces and tension rods, extending from it down to the arch, to which they are also secured. The spaces thus occupied by the bracing are called the spandrels of the arch. We here have the member A B extending over the whole length of the span, sustaining no part of the load, but adding so much weight to the arch. Although it is not subjected to so much strain as the chord in the bow-string truss, Fig. 6, and is consequently much lighter, still, it may be dispensed with altogether, if we divide the material in the arch and place one-half of it a few feet below the other, and thus form two arches with about half the

original material in each, as in Fig. 10, and brace these two arches or ribs thus made in such manner that they will preserve their

form and relative distance from each other, under all circumstances. This is what is usually termed the ribbed arch, and is the form adopted for your Bridge. The roadway above it may be of wood, and can be carried by light struts resting on the arch. We require only a little more material in the arch thus formed to carry a given load than is required in the compressive member alone of the bow-string girder, and we only need the two parts of it far enough asunder to insure sufficient stiffness to resist the strains produced by the partial load. In a 515 feet span, if made of cast steel, eight feet from centre to centre of the ribs is found to be sufficient to sustain the form of the arch when one-half of the span has two tracks covered with locomotives, and the roadway is densely packed with people, the other half of the span being entirely unloaded. The braces for that length of span are only nine feet long, whilst in the bow-string girder of the same span and similar curve of arch, the bracing would be greatly longer, and would weigh four or five times as much. Hence, by using the stone abutments, we save *weight* (and consequently *cost*) in the superstructure, in three ways: firstly, by dispensing with the long, heavy bracing; secondly, by dispensing with the tension member of the truss; and, lastly, by using less material in the arch—for it is plain that, as this member must sustain, in either case, the entire load that crosses the Bridge, it must have more material put in it when it has the heavy bracing and the tension member to support also. In a long span the saving in these three items is really enormous, as will be presently shown. The 515 feet span for this Bridge, with the arch, made of cast steel, weighs about 1,400 tons, exclusive of timber; with the timber, it weighs about 2,000 tons. If the arch were held by a tension member, instead of abutments, that member would weigh about 450 tons, supposing the steel used in it to bear a strain of 20,000

pounds per square inch. But as the arch is only calculated to bear about 3,600 tons, including its own weight, the extra weight of this member would require the arch to be increased in its dimensions by the addition of about 50 tons of steel, making 500 tons in all for one span. At $350 per ton for the steel, this would increase the cost of the span $175,000. The three bow-string girder spans, if made of cast steel, would, therefore, cost over half a million dollars more than the three ribbed arches. They would then weigh 1,500 tons more than the arches, while the saving in the cost of the four piers for their support could not exceed $250,000.

It matters not what truss be used, a proportionate excess of cost over the arch will be found to prevail in every one of them. Where there is no saving in the cost of masonry, or peculiar features of location excluding the *arch*, there can be no substantial argument in favor of the truss for long spans. By the word truss, I include every known method of bridging except the arch. In all of them there must be both a compression and a tension member. In the arch, but one of these two members of the truss is required; the compression member when the upright arch is used, and the tension member when the catenary or suspended arch is adopted. This principle limits the length of the span in trusses by rapidly increasing their cost, so that we will seldom see them used in excess of 350 feet; whereas, the span of the upright or suspended arch may be almost unlimited. If it can be shown that the arch will be as safe and durable, and the entire structure can be made at less cost, and that its form is suited for the location, then there can be no reason why it should not be adopted where a long span is desirable.

This explanation will enable any one to understand, if he will take the trouble carefully to consider it, why an arch for the superstructure is cheaper than a truss.

CAPACITY OF THE BRIDGE.

On consulting your Board as to the capacity which the Bridge should possess, I found it was unanimously in favor of the erection of one that should be capable of accommodating the local trade and travel now existing, or likely to exist for many years to come, between this city and those immediately opposite in Illinois, and at the same time serve for crossing all the trains required by the

ten railways radiating in every direction from St. Louis. To accomplish this it was deemed necessary to provide a carriage-way of sufficient width to admit of four wagons abreast, two foot-ways each eight feet wide for pedestrians, and a double railway track for steam trains.

The accommodation of steam railway traffic and ordinary travel on the same structure, and at the same time, is not an untried experiment. It has been done with entire success on the Niagara bridge, on the high level bridge at Newcastle-on-Tyne, and on several structures of minor note in Europe and America.

To provide a single road-way wide enough to accommodate all of these currents on one level, in such manner as to prevent any annoyance from, or interference with each other, would involve the necessity of a much wider superstructure than if the railways were placed above or below the carriage-way. This wide superstructure would require wider piers and abutments, and thus increase the cost of the entire fabric.

The great number of steam trains to be accommodated by the Bridge makes it absolutely necessary that each track shall be at all times open for their transit, and precludes the possibility of having the rails occupy a road-way to be used, even at stated intervals, by ordinary travel, as is done on some railway bridges where the steam trains cross less frequently. In like manner the great tides of local travel that must constantly occupy the carriage-road, throughout the day and part of the night, preclude its being used in common for steam trains. Hence there would be no alternative if they were all on the same level, but to widen the superstructure to accommodate them. In placing the railways alongside of the carriage road, it would be absolutely necessary that the tracks should be separated from the latter by close partition walls or fences, to prevent the frightening of animals on the Bridge. These walls would increase the weight of the structure, expose a much greater surface to be acted upon by winds, and destroy the attractiveness which the Bridge would possess if it afforded an uninterrupted view of the river and harbor. For these reasons, it was decided not to place the railways on the same level with the carriage road.

The Federal law requiring the lowest part of the Bridge to be 50 feet in the clear, "measured at the centre of the span," above the city directrix, (or ordinary high water,) and the level of the railways on the Illinois shore being but a few feet above the directrix,

it is plain that if the railways of the Bridge were placed above the carriage-ways, it would greatly increase the length of the necessary railway approaches in Illinois. Instead of their being 52 feet above the directrix at the centre of the Bridge, they would have to be at least 70 feet, if they were placed over the carriage-way. This would involve difficulties at the western end of the Bridge also, as the grade would be too great to run the trains under Washington avenue. To have these trains leave or enter upon the Bridge through this crowded avenue, would not be desirable, even if the citizens of St. Louis would permit it. To substitute horse power for steam in moving the trains through the avenue, would cause extra expense, and would not remedy the difficulty, for the street does not possess the capacity to accommodate the railway business as well as the local traffic of the Bridge; and if used for the former, it would be liable to be blocked up at times from the effect of snow storms, accumulation of trade or other causes, and would so interfere with the business of the city and the convenience of the people as to become an unbearable nuisance. The accommodation of railway traffic by the Bridge therefore, if located at Washington avenue, involves the necessity of a tunnel under that street, and the railroad grades on both sides of the river fix the position of the railway tracks on the Bridge, below the carriage-way.

By this arrangement, it was found that the carriage-way would be on the same level with Washington avenue at each end of the Bridge, and it would thus form a continuation of that avenue eastward from Third street 2,700 feet long, entirely level except the slight rise that will be given to the Bridge between the two abutments, to obtain the requisite height over the channel at the centre of the middle span.

The width of the structure and the position of the road-ways being thus determined, the next important step was to decide upon the system that should be adopted on which to construct the Bridge.

The determination to accommodate railway and local traffic on the Bridge, involving as it does the necessity of an upper and lower road-way, increases the magnitude of the structure to such a degree as to render a drawbridge out of the question, even if it were not an absurdity to think of opening and closing, thirty or forty times a day, a highway that must be as constantly and as densely thronged as any street in the whole city. As a drawbridge at this city has been, I think very wisely, prohibited by law, I only allude to it to record my disapprobation of such a structure, if it were considered desirable to obtain a repeal of the restriction. The

objections to the use of one at this location are so numerous, and I think so well understood, that I will not occupy your time in detailing them.

In deciding upon the method of superstructure to be adopted, so much depends upon the difficulties presented in securing proper foundations that it becomes necessary to explain something of the magnitude and character of those difficulties in this case, to enable you to understand fully the reasons which impelled me to the adoption of the arch in spans of about 500 feet.

ACTION OF UNDER CURRENTS.

Examinations made by myself and other engineers have revealed the fact that the bed-rock of the river, which is limestone, is overlaid with a deposit of sand about 15 feet deep near this shore, and perhaps 100 feet at the other; the increase in depth being very regular as we proceed towards the Illinois shore. The borings, as far as made, indicate a regular slope of the rock from this shore, which has been traced as far as the location of the eastern channel pier, where it is about 79 feet below the deposit. Near the Illinois shore, 90 feet of boring has failed to reach the rock. The sandy bed of the river in low water is nearly level.

Soundings made by me prove that this deposit is scoured out to a great depth in time of floods and freshets. Although I have not had any extreme stage of water in which to make my observations, I found that a rise 13 feet less than high-water mark caused a scour of 18 feet. The greatest variation in the height of the river known at this place is about 41 feet. An average depth of about 8 feet, with a width of 1,600 feet, represents the volume of the river at extreme low water at the location selected. Extreme high water covers an immense area of bottom-lands above and opposite the city, and the construction of numerous railway dykes across these from East St. Louis, reduces the water-way at Washington avenue to about 2,200 feet in width at high-water mark. On this shore and on the other, this water-way is thoroughly revetted below the low-water line with rubble stone and protected by the wharf pavements above that line. The concentration into this narrow channel of the vast volumes that are sometimes poured out of the gigantic net-work of streams above St. Louis, the main artery alone of which is navigable over a thousand leagues above this city, assures me that in time of floods it is not improbable that this

deposit is removed to twice or thrice the depth shown by my soundings, and perhaps to the rock itself.

I had occasion to examine the bottom of the Mississippi, below Cairo, during the flood of 1851, and at 65 feet below the surface I found the bed of the river, for at least three feet in depth, a moving mass, and so unstable that, in endeavoring to find footing on it beneath the bell, my feet penetrated through it until I could feel, although standing erect, the sand rushing past my hands, driven by a current apparently as rapid as that at the surface. I could discover the sand in motion at least two feet below the surface of the bottom, and moving with a velocity diminishing in proportion to the depth at which I thrust my hands into it.

It is a fact well known to those who were engaged in navigating the Mississippi twelve years ago, that the cargo and engine of the steamboat America, sunk 100 miles below the mouth of the Ohio, was recovered, after being submerged twenty years, during which time an island was formed over it and a farm established upon it. Cottonwood trees that grew upon the island attained such size that they were cut into cord-wood and supplied as fuel to the passing steamers. Two floods sufficed to remove every vestige of the island, leaving the wreck of the America uncovered by sand and 40 feet below low-water mark, where, in 1856, the property was recovered. Pilots are still navigating the river who saw this wreck lying near the Arkansas shore, with her main deck scarcely below low-water mark at the time she was lost. When the wreck was recovered the main channel of the Mississippi was over it, and the hull of the vessel had been cut down by the action of the current at the bottom, nearly 40 feet below the level at which it first rested; and the shore had receded from it by the abrasion of the stream nearly half a mile.

These remarkable but well attested facts came under my own observation and occurred at Plumb Point, one hundred miles below Cairo, where the Mississippi is more than one mile wide, and where the lateral action of the current is not confined as it is here by stone, and where the depth of the action of the under currents must be much less than at this narrow passage.

Singularly enough, the fact is almost certain that at seasons of *lowest* water, this deposit is also liable to be removed to an extent probably sufficient to lay bare the rock in mid-channel. The current being much less when the water is low, the sand accumulates to its greatest depth. When the river freezes over, which

only occurs when it is quite low, a strong crust of ice, from ten to fifteen inches thick, is formed in this narrow gorge, while there are frequently great stretches of the river above unclosed. The floating ice formed in these open spaces is carried down in large masses, which accumulate in this and other narrow passages of the river, and form what are termed ice gorges. These accumulations sometimes extend several miles above the contracted channels of the river, and cause the water to rise, or in river parlance, "back up," 10 and even 20 feet in some instances, above its former level. The firmly frozen crust serves to hold the masses that are accumulated beneath it, and the great height attained by the "backing up" of the water above the gorge increases the currents that are sweeping below the ice to a degree probably greatly exceeding that of the floods, if we may take the water levels above the gorge as an index to the current created by this hydrostatic pressure. These currents, I believe, would prove too great to be resisted by any ordinary rip-rap (or loose stone) usually used to protect foundations not resting on the rock. The ice being lighter than the water, it follows that these currents will be constantly acting beneath the gorged ice, and in direct contact with the sand. As rapidly as the latter is cut away, fresh supplies of ice are driven under, and thus the mass continues to grow in depth, and the current to be directed nearer to the rock. After a few weeks the pressure of the back-water becomes so enormous as to sweep the gorge away, and on such occasions the open space of water below the gorge is at once filled for miles with the submerged ice thus liberated. This ice can readily be distinguished from the crust or surface ice by its scarcely floating, and by the quantities of sand and mud with which it has been saturated during its imprisonment.

On two occasions I undertook to cut a channel in the ice through which to remove from gorges two valuable diving-bell boats to places of safety. The undertaking was only successful in one case. The surface ice being removed from the canal and hauled off on its sides, I found the quantity of submerged ice which continually arose, when that in sight was removed, was so great that the supply seemed inexhaustible. In the case where I was successful, I was able to cut the channel from an open part of the river up to the vessel, and through it the submerged ice was floated out and the channel thus cleared.

In the winter of 1855, the steamer Garden City, of about 800 tons burden, was inclosed in the ice gorge which formed in this

harbor. Many of our citizens will remember that a partial movement of the gorge caused her sides to be crushed, in consequence of which the vessel filled with water. She was lying at the upper part of the city in front of a large stone quarry, the debris from which had been for several years thrown into the river by the quarry-men, and had formed a steep, rugged shore of such slope as the broken stone naturally assumed. The water where the vessel sunk was 25 feet deep, but she was sustained upon the gorged ice beneath her, so that her deck was scarcely under the surface. She was in this condition when I was called upon to save her. Her hull being about nine feet in depth, it is evident that the ice which sustained her must have been packed to the bottom, and 16 feet deep. This ice supported her with her engines and boilers and a cabin over them about 150 feet long, until the bank was removed, ways placed on the ice under the steamer, powerful purchases secured ashore, and the vessel hauled broadside in to, and upon the bank in safety, before the gorge gave way. The time occupied in doing this was about ten days, nearly all which time the steamer was resting on ice that had been driven under the surface by the action of the current.

The establishment of piers in the channel of the river must facilitate the formation of an ice gorge at the Bridge in the Winter, and they will certainly tend to its retention until the sand is scoured out about and between them to an unknown depth.

For these reasons I have maintained and urged that there is no safety short of resting the piers for your Bridge firmly upon the rock itself. On no other question involved in its construction does my judgment more fully assure me that I am correct, although the Convention of Engineers assembled here last summer announced in their report that they did not consider it essential to go to the rock with all the channel piers of Mr. Boomer's bridge. The Convention assumed that the greatest possible scour would not exceed thirty feet below low-water mark, (equivalent to the removal of twenty-two feet of deposit. See Report, page 80.) I am supported in my opinion upon this matter by many eminent engineers with whom I have exchanged views upon the subject.

The recent destruction of many of the bridges in British India, by having their foundations undermined by the action of floods upon the sandy bottoms of the streams in that country, furnishes a warning that we should not neglect.

MAGNITUDE AND NUMBER OF FOUNDATIONS.

The necessity for basing the channel foundations upon the rock being considered imperative, the next question was to determine the most judicious number of piers.

By shortening the spans the cost of the superstructure would be lessened, and a reduction in the size of the piers be possible. This would, in ordinary cases, result in a proportionate lessening of the cost of the entire structure. The law, however, requires that at least two spans shall each be not less than 350 feet, and the remaining spans 200 feet in the clear. Therefore, the reduction in cost of the structure by lessening the length of spans is limited to a certain extent by this provision. There are reasons, however, involving the safety of the bridge, which make it necessary to provide, by an increase of masonry in the piers, the ability to resist the extraordinary casualties to which they will be liable at this location. When we take into account the great height required for the piers, from the rock to the lower roadway of the bridge, (145 feet in one, and 174 feet in the other,) and remember that by the scour of the current they will be, at times, without the supporting pressure of the sand to resist the strains to which they will be subjected, we have an imperative reason for increasing the size of the piers to a degree sufficient to insure their stability, without reference to any other question whatever.

In another part of this report I have explained why an arch is a cheaper method of *superstructure* than a truss; and that the advantage which the truss has to recommend it, is that it creates no thrust, but simply a vertical pressure upon the piers, thus enabling the latter to be built with less material than when the arch is used. But here we have an absolute necessity for using massive piers, and hence are unable to avail ourselves of the chief feature of economy that might otherwise make the truss available, and the smallest spans permitted by the law desirable. If we bear in mind that the bed rock deepens as we approach the Illinois shore, it will be seen that by adopting shorter spans it becomes necessary to erect piers nearer to that shore, and these must be put down through a greater depth of deposit than the two contemplated in the plan. The deepest foundation we have to put down will be 79 feet below the bottom

of the stream, and we shall probably have an average depth of 20 feet of water during the season that is occupied in placing it in position, making in all 99 feet below the surface of the river. It is certainly not desirable to undertake a deeper one, except for more potent reasons than I can discover in the premises. Shorter spans would make it absolutely necessary to do this, if the piers were placed on the rock. It has been asserted that the erection of 500 feet spans is more hazardous than 350 feet ones. This is admitted; but with ordinary care and judgment the danger of casualties in the erection of either cannot be great. It is certainly more hazardous to erect a structure at this location whose channel piers are not placed upon the bed rock below the river bottom; and if this be done, and shorter spans adopted, the difficulties attendant upon putting down the foundations must necessarily be increased, not only by increasing their number, but also because of the greater depth required for one or more of them. The hazard avoided in erecting the superstructure would simply be added with interest to that attending the construction of the piers.

The magnitude and height of the piers required is so great, even if the spans were lessened and four arches substituted, and the season favorable for their economic erection is so short, extending only from the middle of August to the middle of December, that it would not be advisable to attempt the erection of more than one of them in one year. Hence an additional one would delay the completion of the Bridge, and absorb in interest on capital as much as could be saved by shortening the spans. It is possible to use the same false works and machinery for each pier unless we undertake to put in two of them in one season. This would involve the additional expense of duplicating these works and machinery. But little could be saved in the masonry of the piers by substituting three for the two contemplated, as the three would contain nearly as much as the two. There would be more saved in the abutments, but it would be chiefly in that class of masonry (the backing) which is least expensive. The cost of putting down the necessary false works, removing the deposit, and erecting the three piers, would be considerably greater than for two.

The masonry of your Bridge being so massive in character, has been contracted for at a price greatly under that paid for other large

bridges now being constructed in the West. The reason for this is that the slender piers used in the latter require that every stone in them be cut to an exact size. This is only necessary with the exterior work in yours, and hence it can be and is being executed for about 30 per cent. less than the masonry of those bridges.

The question of relative economy of shorter spans has not been determined by guess work, or decided by judgment alone, but the conclusions arrived at in favor of the spans adopted have been confirmed by careful estimate and calculation to be not only the most judicious, but also the most economical, arrangement compatible with the safety of the structure.

It is not considered advisable to place the abutment at the Illinois shore upon the rock. The cost of doing this would be very great, and as the piles, upon which the masonry will be erected, can be thoroughly protected at the shore from the action of the current, it will not be necessary to do it. The sand on the site of the abutment will be excavated to a point about 25 feet below the bottom of the river, and piles will then be driven to the greatest depth possible, and sawed off a few feet above the sand. The spaces between the piles will be carefully filled, and the masonry laid upon a timber platform on the bed thus formed. The face of this abutment below water will then be thoroughly revetted with stone.

The rock has been laid bare 13 feet below low-water mark on the Missouri shore, and the western abutment commenced upon it within the coffer dam built for the purpose.

METHOD OF SINKING THE PIERS.

A number of designs and estimates were made by me to determine the most practicable, economical and reliable method of constructing the parts of the channel piers below low-water mark. These designs and estimates included the use of cast-iron cylinders, of diameters varying in the different plans from 3 to 15 feet, which were to be sunk to the rock and filled with concrete. The danger of scour, and the difficulty of binding these cylinders together beneath the surface of the sand, so as to insure stability under the strains produced by the thrust of the arches, induced me to increase their diameters in subsequent designs, until they became so great that wrought iron was substituted, and finally two cylinders, each of a

diameter equal to the width of the pier, were tried with smaller ones about them, to complete the entire dimensions of the foundation. The same difficulty of binding these together in a manner to insure safety to the superincumbent masonry, in the event of deep scour, as well as to give promise of any great durability, still remained.

Cast-iron cylinders may be used with great advantage in forming subaqueous foundations in situations where there is no scour, but the dangers to be guarded against in this location would render them, I think, less reliable and more expensive than other methods.

My experience of the effects of fresh water upon wrought and cast-iron, submerged for many years in the Mississippi, assures me that the latter can be relied upon as almost indestructible, but that wrought-iron will oxidize or rust out so rapidly that in twenty years the strength of a bolt an inch and a half in diameter would probably be reduced one-half. To bind these cylinders together, beneath the sand, would greatly increase the cost of adopting them, and to use wrought-iron to secure them above the sand would fail to insure durability. To undertake to do it with cast-iron would be more expensive, and the slightest unequal settlement of the different ones composing the mass would be likely to fracture a material so brittle. To sink these cylinders, either by the pneumatic process or by any of the methods known, to the requisite depth, would be exceedingly expensive. The great quantity of iron required in them, and the fact that they must be filled with masonry, would render a foundation of the necessary dimensions, if composed of them, much more expensive than if made of stone alone.

Having arrived at this point in the solution of the most important problem connected with the design and erection of your Bridge, I determined to construct the base of the pier entirely of solid masonry, within a water-tight floating coffer-dam, whose sides should be extended above water, from time to time, as it sunk deeper and deeper, with its increasing burden of stone and cement.

Piers of smaller dimensions have been constructed in a similar manner, and placed upon foundations favorable to their permanent reception. When sand or mud has been interposed, and its removal rendered necessary, the sides of the floating vessel have been extended downward below its bottom, to form a chamber or kind of

diving-bell beneath the masonry. Through the masonry, tubes were provided by which workmen and materials could descend into the chamber, and through these tubes air was forced to expel the water from the chamber, and enable the workmen to remove the sand or mud beneath the pier. These tubes required to have two or more air-locks or valves in them, that were closed behind the workmen or materials in their passage, to prevent the escapement of the compressed air in the chamber. This of course retarded the rapid progress of the work. To facilitate the excavation of the deposit an extra tube was introduced in the middle of the pier and extended to the level of the bottom of the air chamber. The water stood within this tube at the level of the surface of the river, and through it an endless chain, carrying scoops or excavators, was made to rotate around a pulley at the bottom of the tube, and another at the top. In this way the sand was rapidly excavated without permitting the escapement of air from the chamber, and without passing the deposit up through the air-locks. The workmen in the chamber were enabled to shovel it to the bottom of the tube, where it was taken by the excavator, and discharged in vessels above.

The gradual descent of the pier was managed by screws, supported upon false works, erected around and over the site of the pier. As the sand was removed below, the pier was allowed to settle by slacking the screws, as it was only partially water-borne. When it had passed through a considerable depth of sand, the friction of the latter, upon the sides of the pier, held it to such a degree as to take all the strain off the screws, and when it moved downward it was sometimes so suddenly that the supports were strained severely.

The shortness of the season in which each one of the piers for this Bridge must be put in position, because of the floods of Summer and the ice of Winter, and the great amount of deposit to be removed, renders the pneumatic process just described too slow for this case, as well as too expensive. For the safety of the workmen beneath the pier, it is absolutely necessary to regulate its descent by screws or similar means, and to do this with piers of such magnitude would not be advisable.

The removal of the sand will be accomplished by sinking an elliptical-shaped caisson or curb of plate-iron through the deposit to

the rock. This caisson will be open at top and bottom, and will be strongly braced on the inside with heavy angle irons placed horizontally around it. It will be larger at bottom than top, to facilitate its passage through the sand and relieve it of the friction. The caisson will be suspended by false works erected around the site of the pier, and will be regulated in its descent by screws supported on the false works. As it is lowered into the sand, that which is inclosed by it will be excavated by steam machinery, until the caisson is finally sunk to the rock. It is not intended at any time to remove the water within the caisson, but only the sand it incloses; the object of the caisson being only to exclude the sand outside of it until that which it incloses has been removed, the rock leveled off with concrete, the floating coffer dam placed in position within the caisson, and the pier so far built up in the latter as to sink it down to the concrete bed prepared for it.

The bottom of the coffer dam will be formed of squared timbers, thoroughly caulked, and will be about two feet in thickness. Its sides will also be of timber, and so constructed as to admit of being disengaged from the bottom when the latter has reached the bed formed to receive it. The interior of the coffer dam will be larger than the pier, and the latter will be constructed with certain cavities in it to be filled with masonry after the pier reaches the bottom, so that the weight of the pier will bear such proportion to the displacement of water as to insure the top of the masonry being kept but little below the surface of the river while the pier is being built within it. This will enable the sides of the vessel to be thoroughly braced against the pier, so as to resist the pressure of the water.

It is known that timber is indestructible when completely submerged in fresh water. Piles placed in the Rhine by the Romans, nearly 2,000 years ago, have been found to be entirely sound when removed within the present century. There are many other similar instances on record establishing the fact of its durability, whilst the soundness of the timber found in the bogs of Ireland and elsewhere indicates that it is unlimited by time.

When the bed rock has been prepared to receive the pier, the coffer dam will be floated within the caisson, and will be guided by the latter as it descends with its load. It will be understood that the pier is completely water-borne by the coffer dam until the quantity of

masonry in it has become so great as to cause the dam to touch the bed on which its bottom, with the pier, is to rest permanently. When the pier has been completed above water, the dam is permitted to fill, and its sides will then be disengaged from the bottom and removed, to be used in putting down the next pier. The caisson for the smaller pier can be withdrawn and used for the other one; and the larger one may possibly be saved also.

As before stated, the floating coffer dam is not an untried experiment, but has been frequently used to place piers in position where the bed-rock or other substratum was favorable for their reception. The caisson has also been frequently used to exclude the sand or mud, and enable that within it to be removed sufficiently to facilitate the driving of piles to a greater depth and in firmer soil than would be otherwise practicable.

The estimates made for the cost of this work prove that it will be much less expensive than any other method yet devised; while the superiority of the foundations thus made will be beyond all question.

PLAN OF STRUCTURE.

The Bridge will have three spans, each formed with four ribbed arches made of cast steel. The centre span will be 515 feet and the side ones 497 feet each, in the clear. The rise of the centre one will be one-tenth of the span; that of the side ones forty-seven feet ten inches each.

The four arches forming each of these spans will each consist of an upper and lower curved member or rib, extending from pier to pier. Each of these members will consist of two parallel steel tubes, nine inches in exterior diameter, placed side by side. The upper and lower members will be eight feet apart, measured from the centre of the upper to the centre of the lower tubes. At regular intervals of about nine feet, these members will be braced from each other by a vertical system of cast steel bracing on each side of them. These braces will be secured at each end to cast-steel plates, formed something like the voussoirs of a stone arch, and against which the tubes will be abutted and secured every nine feet throughout the arches. A horizontal system of bracing will extend from pier to pier between the four upper curved members, and a similar system between the

four lower ones, for the purpose of securing the four arches in their relative distances from each other, and to sustain them against lateral pressure.

The two centre arches of each span will be thirteen feet nine and a half inches apart from centre to centre, and will have, in addition to the upper and lower horizontal bracing just described, a system of diagonal bracing, securing the upper member of one arch to the lower one of the other arch, and the two other members in like manner. The outside arches are each fifteen feet one and three-quarter inches from the middle ones, and are joined to the latter by three systems of bracing similar to those described as between the two centre arches. These systems, however, on the outside of the middle arches, extend only from the piers to the under side of the railways, the latter being carried between the two outer and the two inner arches near their crowns. The outside arches being supported in this interval against lateral movement, by rigid connections from both the upper and lower roadways.

The roadways are formed by transverse iron beams, 12 inches in depth, supported by iron struts of cruciform section resting on the arches at the points where the vertical bracing of the latter is secured. That portion of the railways which passes below the crown of the arches is suspended from them. Between the iron beams forming the roadways, four parallel systems of longitudinal wooden members are introduced, extending from pier to pier and serving to maintain the iron beams in position. These wooden members are each about nine feet long, and their ends rest upon the flanches of the beams, and are there secured from moving. On these, the wooden beams for the carriage-way rest in one roadway, and the cross-ties for the railways in the other. From the opposite ends of the iron beams a double system of diagonal horizontal iron bracing serves to bind the whole together, and gives additional support against wind pressure.

The upper roadway is 34 feet wide between the footwalks. The latter are each eight feet wide, making the Bridge 50 feet wide between the railings.

The railway passages below the carriage-way will each be 13 feet 6 inches in the clear and 18 feet high, and will extend through arched openings of equal size in the abutments and piers.

The railways will be carried over the wharfs on each side of the river on five stone arches, each 26 feet wide, and will be inclosed throughout this distance by a cut stone arcade of twenty arches supporting the upper roadway. After passing over those stone arches, the railways will be carried through the blocks between the wharf and Third street on brick arches into the tunnel at Third street and Washington avenue. Over the intervening streets they will be carried on wrought-iron trusses.

On the Illinois shore the railways will curve off to the north and south immediately after crossing the last one of the stone arches, and with a descending grade of one foot in one hundred, extending about 3,000 feet, and supported on trestle work part of the way, they will reach the grade of the railways in East St. Louis.

The carriage road will begin to descend with a grade of five feet in one hundred, at the eastern end of the bridge, immediately after the railway tracks curve away from the latter, and will conform at Third street, in East St. Louis, to the grade of that street. On the Missouri side the carriage-way will be continued over the railway tracks from the Bridge to Third street on a level grade.

MASONRY.

The greater part of the stone for the Bridge will be taken from the Grafton quarries, on the Mississippi, in Illinois, about 40 miles above St. Louis. This stone is a magnesian limestone, of fine firm texture, yellowish in color, and is found in regular strata, varying from one to three feet in thickness. From severe chemical tests, and the proofs of its durability given in many of the large buildings in this city constructed with it, it is believed to be well suited for the intended purpose. It will not, however, be used on the exterior of the work above water. From two feet below low-water mark to two feet above high-water mark, the exterior of the piers, including those on the wharfs as well as the abutments, will be of the best quality of granite. This will be laid in courses not less than thirty inches thick, with an arris cut around each block to indicate the joints of the work, while the remainder of the block will retain the quarry or rough face upon it. Above the granite, the exterior will be entirely of cut sandstone. A granite course, eight

feet in thickness, will be laid through the channel piers, and in the abutments, to receive the skew-backs or heavy cast-iron plates, against which the ends of the arches will rest.

STRENGTH OF THE BRIDGE.

The arches have been designed with sufficient strength to sustain the greatest number of people that can stand together upon the carriage way and foot paths from end to end of the Bridge, and at the same time have each railway track below covered from end to end with locomotives. With this enormous load the strength of the arches will be taxed to the extent of less than one-sixth of the ultimate strength of the steel of which they will be constructed. The piers and abutments have been designed with a view to sustain either span when thus loaded, even if the others were entirely unloaded, and to sustain either span entire if from any cause the adjoining ones should be destroyed. The arches have also been designed to resist the effects of any portion of the span being loaded, as above stated, with any other portion of the same span entirely unloaded.

It will be seen, therefore, that the Bridge has been designed to sustain a greater load than will ever be placed upon it. No occasion can possibly occur requiring it to be densely packed with human beings on the upper road way, and at the same time have its railways covered with locomotives below. Yet the ultimate strength of the materials of which it will be composed is such that the three arches are capable of sustaining twenty-eight thousand nine hundred and seventy-two tons, before they would give way under it.

The superabundant strength of the piers to resist the effects of ice, and the ability of the superstructure to withstand the most violent tornadoes, is clearly demonstrated in the appendix to this report.

It has been asserted by some of your opposers that the pressure of such long arches upon the abutments would be so great that the stone would crush under the effect of it. The ridiculous absurdity of this statement is exposed by the fact, that one block of ordinary lime-stone, six feet square, requires a greater load to crush it than the heaviest burden that can possibly be imposed upon all three of the

arches of your Bridge with the entire weight of the three spans themselves added to it. The weight of the three spans, and the maximum load they are designed to bear, is seven and two-tenths tons per lineal foot, or 10,865 tons. The six feet cube of limestone will require 5,000 lbs per square inch, or 12,960 tons to crush it. I have already stated that the thrust of the piers would be taken upon granite courses, eight feet thick. Granite being more than twice as strong as limestone, the absurdity of this notion is still more apparent. As the thrust of each end of each arch will be received on a surface of granite equal to twenty-four square feet, and as each span has four arches, it follows that the thrust of the three spans is taken on a surface of 576 square feet of granite. This would require, at 10,000 lbs. to the square inch, four hundred and fourteen thousand seven hundred and twenty tons to crush it, proving that of all the ridiculous assertions advanced, this is the most extravagant. It could only be excelled by the fear that the tremendous thrust of these 500 feet arches will prove so great as to force the banks of the stream asunder, and let the fabric into the abyss through which the mighty river would then escape.

EFFECTS OF TEMPERATURE.

By making the longitudinal members of the roadways of wood we avoid the expansion and contraction of those long level platforms between the piers, and leave them to be effected only by the action on the arches.

The longest of the arches will rise at the centre a little less than eight inches by the expansion of the steel when under the greatest extreme of heat, and with the most intense cold it will fall as much below the point at which it will be maintained under a medium temperature. This supposes a range of temperature from 20 degrees below zero to 140 degrees above. (Fahr.) In the appendix it will be seen that in determining the size of the several parts of the arch, we have duly considered the strains resulting from this change of form. The severest strain produced by temperature occurs at the abutments, and does not amount to over four tons per square inch.

The effect on the roadways is simply to raise and lower them at the centre of the arches so imperceptibly that the eye could not de-

tect it. The rise and fall of the roadways of the Niagara bridge is stated by Mr. Roebling to be two and a quarter feet at the centre of the span, under a change of 100 degrees of temperature. That is, the roadways are two feet three inches higher at zero than when the thermometer marks 100 degrees. No inconvenience has been found to arise from this change of form in that bridge, and none can be apprehended from it in yours, where it will be so much less.

There is no truss combined with the arch in the method adopted. The roadways being simply carried on vertical supports by the arches, form no part of a truss system. The arches are the sole supporting members of the structure, and by the vertical bracing between their upper and lower parts, are made amply rigid to sustain their burdens without the use of a truss in combination with them.

I am thus particular in stating this fact, as one of the misapprehensions existing in regard to the plan adopted by you is, that it is a combination of the arch and truss. The well known difficulties caused by the unequal expansion and contraction of each, when the two systems are combined in a metal structure of long span, would naturally create a want of confidence in your plans, if this impression were permitted to prevail.

TUNNEL.

The centre line of the bridge strikes the eastern side of Third street, fifty-three feet north of the northeastern corner of Washington avenue and Third street. At this point the tunnel, which at present has been designed for the accommodation of a single railroad track only, begins. It follows Washington avenue to Ninth street, and by a curve extending through blocks 172, 281, and 280, reaches Eleventh street. It is carried under Eleventh street to Chestnut, when the offset at Market street necessitates another slight curve. After reaching Eleventh street, beyond Market street, the tunnel is carried under Eleventh street to its termination a few feet beyond Clark avenue. Its entire length will be 4,800 feet. The railroad thence is carried to its junction with the Pacific, the Iron Mountain, and the Southwest Pacific Railroads through an open cut.

The width of the tunnel is fifteen feet; its height from top of rail to crown of arch is seventeen feet. The grade in the tunnel is fifty-

five feet per mile, for a distance of 1300 feet, while the remainder has grades varying from ten to twenty-two feet per mile.

The depth of the exterior crown of the arch, below the grade of the street, varies from two to ten feet. The sewers intercepted by the tunnel are not very large ($2\frac{1}{2}$ by $3\frac{1}{2}$ feet on Washington avenue, and $3\frac{1}{2}$ by $4\frac{1}{2}$ feet on Seventh street). New sewers carried alongside the tunnel will provide for the drainage of those intercepted.

Water and gaspipes will be carried through the crown of the tunnel.

The apprehension felt by some persons that in the construction of this tunnel rock and quicksand would form a source of expense and danger, has been set at rest, since borings made under my directions at points about 300 feet apart, throughout the whole length of its proposed centre line, have shown that the only material to be removed is blue and yellow clay.

The tunnel is to be built in sections, and by open cut, and the right for its construction has been granted by the City Council.

WORK DONE.

Under this head I have to report, among other items, the construction of the coffer dam for the western abutment, the excavation of the material inside of the dam to the solid rock, thirteen feet below low water of December, 1863; and the laying of 1,040 cubic yards of the masonry of this abutment. The work on the abutment was stopped on account of high water, after being carried up to an average height of twelve feet, about the 15th of March, and the river has not yet receded enough to allow of a resumption of the work.

Great difficulties were encountered in the construction and maintenance of the coffer dam, by the large number of wrecks of steamers and barges found in that location, which formed an almost impenetrable mass of timber and iron. Parts of three steamboat wrecks, and four barges imbedded in about twelve feet of debris, were encountered and removed within the dam. The work now however is beyond any danger, and may be resumed and carried out without interruption as soon as the river recedes a few feet more.

The large frame-work and the machinery for laying the stone, designed to expedite the construction of the channel piers, has been

completed and erected over the western abutment, to have the machinery fairly tested and its manipulation fully understood before using it on the piers, where so much depends on the celerity of operations. The machinery is driven by one engine, and is capable of placing 500 tons of stone in position in ten hours. Its performance gives entire satisfaction.

The work on the western abutment was carried on under the superintendence of Mr. Benj. R. Singleton, C. E., who deserves great credit for the zealous performance of the duties intrusted to him.

Since stopping the work on the abutment, the foundations for two of the piers of the arches which carry the roadway across the levee, amounting to 433 cubic yards of masonry, were constructed. There is also on hand a large quantity of cut and undressed stone and cement ready for a resumption of the work.

Under this head should also be mentioned the large amount of office-work performed in the calculations and plans for the Bridge. The whole project, in all its details has been thoroughly studied, and but little more remains to be done in that line.

CONVENTION OF ENGINEERS.

The organization of two companies about the same time for the purpose of bridging the river at St. Louis, and the rivalry existing between them for nearly twelve months prior to their consolidation under the present organization, was the cause of many difficulties thrown in the way of the construction of your Bridge. One of these companies, generally known as the Boomer Company, called together a Convention of Engineers last August, to consider the question of bridging the Mississippi river at this point. Although composed in part of many distinguished and able engineers, it was known to have been held solely in the interest of that Company. The plans designed for that Company by Mr. S. S. Post, C. E., were laid before it and approved by the Convention. The plans designed for your Bridge, and adopted by you, were not solicited by the Convention for its examination, and at no time were those plans under consideration by it. At no time did the Convention take up the subject of bridging the river by *arches*, but simply by

trusses, and it therefore very properly recommended that no spans exceeding 350 feet in the clear should be adopted.

Notwithstanding all these facts, it was industriously reported by your opponents that the plans adopted for your Bridge had been condemned by that Convention as unsafe, enormously extravagant, and utterly impracticable; and that it had also condemned the location of it as very injudicious. It is because these statements are even yet repeated by parties interested in defeating the erection of the Bridge, and because they are credited by many persons really anxious for its completion, that I call your attention to them, and pronounce them one and all utterly untrue. One effect of these misrepresentations has been to create a belief in the minds of many, that the plans adopted by you will involve a much greater outlay than is really necessary. This impression has been strengthened, no doubt, by the fact that those plans represent piers and abutments much more massive, and a superstructure far more graceful and elegant, than any form of truss bridge yet constructed. Yet one of the most beautiful and graceful structures in this or any other country, with its massive masonry and enormous span, is one of the cheapest ever erected. I refer to the suspended arch bridge of Roebling's at Niagara.

We are too prone to associate our contemplation of the beautiful in architecture and engineering with an idea of costliness, which is not always just. It is easy to prove, beyond the possibility of a question, that in no other form could the material in those members of your Bridge which impart to it the chief feature of its gracefulness, be used with such economy.

It rarely occurs that any great enterprise is undertaken and completed without some opposition, no matter how praiseworthy the purpose, or how many millions will be benefitted by the work. If the private interests of some one or more individuals are affected by it, opposers, both open and secret, will be on the alert to assail it, and delay or defeat its consummation. It would be strange, indeed, if your undertaking met with nothing but encouragement, and proved an exception to a rule that is, unfortunately, almost invariable. From its inception, you were opposed by a rival Bridge Company, whose antagonism was stimulated and encouraged by the active or passive co-operation of members of two wealthy monopo-

lies, (the Ferry and Transfer Companies,) and by others, actuated by motives best known to themselves.

The consolidation of the two Bridge Companies has removed the rivalry between them, and every legal doubt as to your chartered privileges also; but the opposing influences of the Ferry and Transfer Companies remain. It would have been wonderful if the plans of your Bridge should have escaped, not only severe criticism, but unjust misrepresentation also, in a controversy that has prominently occupied the attention of the public for several months past, and elicited great warmth of feeling. In this controversy, the safety of the Bridge has occasioned much discussion, and the most ridiculous assertions on this point were again and again repeated. Originating with those who have opposed the erection of the Bridge, these objections have been adopted, in some instances, by men really anxious for your success, but who have not had the time, or felt sufficient personal interest in the matter to investigate it for themselves. Others again, occupying the position of wealthy citizens, have perhaps felt the necessity of some apology for not aiding an enterprise so commendable, and have willingly adopted these misrepresentations to excuse their own indifference. Instead of generously abstaining from placing obstacles in the path of an enterprise that should command the best wishes of every one, they have in this way aided its most determined opposers.

The effort to create a want of confidence in the safety of your Bridge was supported, to a certain extent, by the fact that this Convention declared in its report that there was no engineering precedent for a span of five hundred feet, and also by stating that "there has been no bridge of the character of that which (in our judgment) is required at this place yet constructed, to furnish us with any reliable and certain data on the serious questions of materials and workmanship in spans of such great length."

By reference to a copy of the official publication, in my office, made by the Dutch Government in January, 1866, of the details and plans of the Kuilinburg bridge over the Leck, an arm of the Rhine in Holland, you will see that its greatest opening is spanned by a truss of 157 metres, or 515 feet in length, constructed on the method used in the bridge at Hartford, Connecticut. This bridge

has a double-track railway through it, and this truss weighs nearly 2,400 tons, and is partly of steel. (*See wood cut, page* 72).

In 1801 the great Scottish engineer, Thomas Telford, proposed to replace the old London Bridge with one of cast iron, having a single arch of *six hundred feet* span. His suspension bridge over

Telford's proposed bridge of 600 feet span.

the Menai Straits is one of the most substantial structures of the kind in the world, and spans 570 feet. A cast iron arch bridge of a single span of 500 feet was proposed by him in preference to the suspension one, but was rejected by the government, because the arch gave less room on each side of the channel for sailing vessels.

Telford's proposed 500 feet bridge for the Menai Straits.

For forty years this remarkable man continued to enrich Scotland and England with some of the most stupendous and successful triumphs of engineering skill to be found in Great Britain. The erection of more than 1,200 bridges by him, many of them of cast iron, made his experience in bridge construction, superior to that of any man of his period. Many of those erected by him are among the largest and most substantial structures in that country.

A select committee was appointed by Parliament to examine his plans for the 600 feet arch, and the opinions of the most eminent, practical, and scientific men of the British Empire were

taken before it on the subject, among whom were James Watt; John Rennie; Professors Robinson and Playfair of Edinburgh, and Hutton of Woolwich. The plans were approved and adopted, and the work upon this stupendous arch was actually begun.

Although this great work was ultimately abandoned, it was from no want of confidence in the plan, but because (according to Stephenson) the height of the arch (65 feet) involved the necessity of raising the streets leading to it, by which too much valuable property would have been depreciated. In a private letter to a friend, Telford informs him that his plans were adopted for this bridge, and says: "If they will only provide the means, and give me elbow room, I see my way as clear as mending the auld brig at the burn."

Surely, the recorded judgment of such a man as Telford, when sustained by the most eminent men of his day, asserting the practicability of a *cast iron* arch of 600 feet span in 1801, furnishes some "engineering precedent" to justify a span of 100 feet less in 1867.

When we take into account that the limit of the elastic strength of cast iron in compression is only about 8,000 lbs. to the square inch, and that in cast steel it is at least seven or eight times greater; and consider the advance that has been made in the knowledge of bridge-building since the days of Telford, it is safe to assert that the project of throwing a single arch of cast steel, *two thousand feet* in length, over the Mississippi, is less bold in design, and fully as practicable, as his cast iron arch of 600 feet span. Engineering precedents have nothing to do with the question of length of span in a bridge. It is a money question altogether. The problem to be solved is simply, what length of span will pay best? This being decided, and profit enough assured to justify the outlay, engineering skill and knowledge will be found fully equal to its accomplishment, no matter what may be the length required. That one made of a material eight times as strong as cast iron, is unsafe or impracticable 500 feet long, is almost too ridiculous to be noticed, in a country where the assertion is rebuked by Wernwag's *wooden* arch of 340 feet, which spanned the Schuylkill at Phiadelphia.

It must be remembered that the report of the Convention has the names of several able engineers appended to it who were not present

at its meetings; that those who were present considered no method of construction except trusses; that its deliberations for the solution of the grave questions involved in bridging this river occupied scarcely ten days; and that it was convened almost solely in the interest of Mr. Boomer, who then controlled one of the charters of your consolidated company, and the patent for the truss bridge he intended building. When these facts are considered, in connection with each other, it will be understood why the plans for your Bridge were not solicited for comparison with Mr. Post's patent truss; and when you take the statement of Mr. Post himself, as chairman of the committee on superstructure, that to span a clear opening of 500 feet with his truss, would cost as much as to span two openings of 350 feet each, and one and a half of 250 feet each, in addition; or that the superstructure of the Bridge, if built on his plan, would cost $750,000 more with one 500 foot span than if two of 350 feet were used, you will understand fully why the preference was given (on the score of economy) to spans of 350 feet.

An investigation by the Convention of the plans adopted by you, would have revealed the fact that the superstructure of your Bridge, possessing greater strength than the one it endorses, and with its great openings, could be erected for about four hundred thousand dollars less than the truss bridge approved by them, with its greatest spans of but 350 feet; and no part of this saving is absorbed by cost of foundations, as those approved by it, on account of the great quantity of iron required, are more expensive also than yours. The proof of these facts will be found in another part of this report, and they are set forth in a manner that admits of no refutation.

It is however not so easy to understand why a body so intelligent as this Convention, should forget the authority of Telford and his eminent cotemporaries, and the 500 feet truss bridge over the Leck at Kuilinburg, in Holland, and be led into the error of asserting that there was no "engineering precedent" for a span of 500 feet. If it were expected to span this river with an exact copy of some bridge now standing elsewhere, the necessary data could be obtained and applied without convoking so much ability. Any respectable bridge building firm in the country has, no doubt, sufficient engineering talent constantly in its service for such an emergency, and could have had the requisite plans copied, and the

structure erected, without calling a Convention of such distinguished gentlemen to deliberate upon them. Where *no* "engineering precedent" exists however, and where data "on the serious questions of materials and workmanship in spans of such great length" *are not* supplied by structures of equal magnitude, there *is* a necessity for bringing to the consideration of the subject the profoundest thought, based upon such a thorough acquaintance with the strength of materials as experience and experiment alone can furnish, together with a knowledge, obtained by careful study and observation, of the laws which guide us in the combination of those materials.

For increasing the dimensions of a truss beyond any now existing, a knowledge of the strength of materials, and the laws that govern their application, was sufficient to enable the convention to deduce with entire safety, such data from the experience furnished by the 450 foot truss of Brunel, over the Tamar, the 397 feet trusses of the Dirshau Bridge, over the Weichsel, and a dozen others of lesser span, if the Kuilinburg truss were not in existence.

The wording of the report, inconsiderately, and I believe quite unjustly to the members of the Convention, makes that body seem to condemn the adoption, not simply of a *truss* of 500 feet, but a *span* of that length, whereas it really investigated no other methods of construction to determine their relative economy with the truss. It simply compared the 500 and the 350 feet trusses with each other; and instead of being content to condemn the use of the long one on the score of economy alone, which would certainly have been sufficient, it thoughtlessly gives a reason for not using a 500 feet span that is not only unsupported by truth, but which is also a discreditable one to a profession whose greatest merit lies in its ability to overcome difficulties by the application of physical laws, *without* the aid of precedents. By this negligent (or adroit) wording of the report, the professional reputation of the members is made to injure a kindred enterprise of whose existence they were not ignorant, by making each one of them appear to condemn the plans of a rival structure they had never seen. A thing which no one of them would do deliberately, if he valued his own reputation.

The biographer of Telford relates that a scheme for a broad ship-canal was started to connect the Mersey, opposite Liverpool, with

the estuary of the Dee, the object being to enable shipping to avoid the shoals and sand banks that obstruct the entrance to the Mersey. Telford entered on the project with great zeal, and his name was widely quoted in connection with it. It appeared, however, that one of its projectors, who had secured the right of pre-emption of the land on which the only possible entrance to the canal could be formed, suddenly sold out for a large sum to the corporation of Liverpool, who were opposed to the plan. His biographer says that "Telford, disgusted at being made the instrument of an apparent fraud upon the public, destroyed all the documents relating to the scheme, and never spoke of it afterward, except in terms of extreme indignation."

Considering that the Convention was assembled solely in the interest of a rival company, and after the fact of your adopting 500 feet spans had been published, the inference drawn from this part of the report is quite conclusive that the eminent reputation and distinguished standing of its members, have been used for a purpose quite similar to that related of Telford; and knowing that the same keen regard for rectitude displayed by that engineer, is shared in by almost every member of a profession based on laws incapable of deception, and the daily application of which, in the routine of their duties, naturally inculcates a love of all that is truthful and correct; I feel assured that they have cause to feel, and doubtless do feel, equally indignant with Telford.

If there were *no* engineering precedent for 500 feet spans, can it be possible that our knowledge of the science of engineering is so limited as not to teach us whether such plans are safe and practicable? Must we admit that because a thing never has been done, it never can be, when our knowledge and judgment assure us that it is entirely practicable? This shallow reasoning would have defeated the laying of the Atlantic Cable; the spanning of the Menai Straits; the conversion of Harlem Lake into a garden; and left the terrors of the Eddystone without their warning light. The Rhine and the sea would still be alternately claiming dominion over one half of the territory of a powerful kingdom, if this miserable argument had been suffered to prevail against men who knew, without "an engineering precedent," that the river could be controlled, and a curb put upon the ocean itself.

COMPARISON OF COST.

By the consolidation of the two Companies, you have been placed in possession of the estimates for the Bridge designed for Mr. Boomer, and approved by the Convention of Engineers. I propose to compare those estimates with the estimated cost of your Bridge, taking the same prices in each case for the same materials, as far as practicable.

Comparison of the Cost of Construction of a Truss of 500 *feet span, with that of a Ribbed Arch of the same span, (for equal loads).*

The cost of a 500 feet truss is not directly stated in the proceedings and report of the Board of Civil Engineers, but on page 48 of that report we are informed that it was proved in joint committee that "the cost of a span of 520 feet (or 500 feet in the clear) will be more than twice as great per foot of Bridge, as a span of 368 feet," and being in possession of Mr. Post's estimate of cost for a truss of 364 feet span, we can easily deduce the cost of a 500 feet span.

The following is a copy of Mr. Post's estimate:

COST OF ONE SPAN OF 364 FEET—(FOUR TRUSSES).

Cast Iron	1,732,992	lbs @	7	cents ₩ lb		$121,309 44
Wrought Iron	1,679,918	"	12	"		201,590 16
Rolled Iron	240,775	"	9	"		21,667 95
Railing for Sidewalks	56,056	"	15	"		8,408 40
Spikes, Bolts, etc	19,656	"	6	"		1,179 36
Trams	65,022		10	cents		6,502 20
Pine Timber	163,200	feet	8	"		13,056 00
" for protection	14,560	"	4	"		582 40
Oak Timber	34,000	"	10	"		3,400 00
Scaffolding and Raising						36,800 00
Painting						8,000 00
Patent Fees						6,956 04
Testing and Engineering						42,945 19
Contingencies						118,099 28
Total						$590,496 42

Or, $1,622 per lineal foot.

The estimate for the whole superstructure, of which the above is a part, amounts to $3,233,627 82. As the total cost of superstructure, given in page 84 of the Proceedings and Report of the Board of Civil Engineers, including the cost of 6,000 feet of rail-

way, amounts to $3,638,920, or nearly $400,000 more, it is evident the above cost of the span is not overstated, and it must be plain that if any error is committed in this comparison, it is in favor of the truss bridge.

The cost per lineal foot of bridge of a truss of 364 feet span being $1,622, the cost of a truss of 520 feet would be more than twice as great per lineal foot, according to the above extract, or $3,244 per lineal foot, or $1,686,880 for the whole span.

If we introduce the same prices for the same materials, used in this estimate, into the estimate for the ribbed arch of 515 feet span, as far as applicable, and allow the same per-centage for contingencies, we obtain the following comparative estimate of cost of superstructure for an arch of 515 feet clear span:

Cast Steel	848 tons in arch	$320 00	$271,360 00
Wrought Iron	158 "	240 00	37,920 00
Rolled Iron	374 "	180 00	67,320 00
Cast Iron	61 "	140 00	8,540 00
Cast Iron in Roadway	7 tons	140 00	980 00
Spikes, Bolts, etc	34 "	150 00	5,100 00
Pine Timber	196,000 feet	8	15,680 00
Oak Timber	23,000 "	10	2,300 00
Nicholson Pavement	191 squares	35 00	6,685 00
Galvanized Iron Lining	26,500 sqr. feet	20	5,300 00
Railing	1,060 lineal feet	8 00	8,480 00
Cornice	1,060 "	1 50	1,590 00
Painting			12,000 00
Raising			20,000 00
Testing and Engineering			45,000 00
Contingencies, 25 per cent			127,063 75
Total			$635,318 75

Or, $1,233 per lineal foot of Bridge.

The cost of a ribbed arch of 500 feet span would therefore be $616,500, or $1,070,380 less than the cost of a truss of the same span on the plan approved by the Convention.* It may be said that

* I have no doubt that the substitution of steel in the upper and lower members of the truss, and in its struts and braces, would make this comparison more favorable for the truss, but it would still be greatly in favor of the arch. I give the figures however as used in estimates made for the actual construction of the two systems; by Mr. Post for the one, and by myself for the other, when there was no probability of the two estimates being used in comparison.

The proof of the greater economy of the ribbed arch over the truss in superstructure, as explained in *this report* under the head of *Arch and Truss*, is clearly demonstrated by this comparison, though it is proper to state that the portion of the report under that heading was merely designed as a general illustration of the principles involved, without aiming at mathematical accuracy. In the Appendix the investigations and calculations for the Bridge are set forth in a manner to challenge the most critical scientific analysis.

the prices in the truss estimate are higher than those at which the work could be executed. This may be true, but on the other hand the estimate for the ribbed arch here given is $154,263 more than the actual estimate of its probable cost to be found in another part of this report, and which is based on proposals from reliable parties. By the latter estimate its cost per lineal foot is $934.09.

Comparison of the Cost of Superstructure (from Levee to Levee) of the Bridge, approved by the Board of Civil Engineers, with the Cost of Superstructure of the Arch Bridge:

The Bridge approved by the Board of Civil Engineers consists of two trusses of 368 feet each, and four trusses of 264 feet each (from Levee to Levee).

The cost of a 364 feet truss, as before stated, is $1,622 per lineal foot, and the cost of a truss of 244 feet span is, according to the same estimate from which the above figures are quoted, $243,805 51, or $1,000 per lineal foot. Even assuming that the cost of a truss of 264 feet span (for which we have no estimate) would be the same per lineal foot as that of a truss of 244 feet, while it evidently would be more, the cost of superstructure of the truss bridge (between levees) would then be:

2 Trusses of 368 feet ($1,622 per lin. foot)	$1,180,992 84
4 Trusses of 264 feet ($1,000 per lin. foot)	1,056,000 00
Total cost of superstructure	$2,236,992 84

Your Bridge requires, between levees:

1 Span of 515 feet ($1,233 per lin. ft.)	$ 635,318 75
2 Spans of 497 feet ($1,233 per lin. ft.)	1,225,602 00
Total cost of superstructure	$1,860,920 75

or nearly $400,000 less than the truss bridge approved by the Convention.

If, instead of the two trusses of 368 feet span, and four trusses of 264 feet span, one truss of 500 feet span and five trusses of 244 feet span each had been adopted, the cost of superstructure would have been as follows:

1 Span of 500 feet	$1,687,000 00
5 Spans of 244 feet (1,000 per lin. ft.)	1,220,000 00
Total cost of superstructure	$2,907,000 00

or $670,000 more than if built on the plan recommended by the

Board of Civil Engineers, and $1,046,000 more than on the plan adopted by you.

That the cost of a 500 feet truss is not stated too high in the above comparison, is proved by the statement, contained on page 43 of the Report of the Board of Civil Engineers, that the construction of the truss bridge, with a span of 500 feet, would involve an extra cost of about three-fourths of a million of dollars ($750,000), while, according to the figures adopted in our comparison, the difference is $670,000 only.

I have no means of knowing the exact estimate for the substructure of the truss bridge, but it is stated, on page 82 of the Report, at $2,541,007. The estimated cost of the substructure of your bridge, including its approaches, is $2,060,477.24.

SUSPENSION AND UPRIGHT ARCH BRIDGES.

Although great difference of opinion exists among engineers as to the safety and durability of suspension bridges, and also as to their fitness for railway purposes, I believe they very generally, if not unanimously, agree that for long spans, that method of bridging is of all others the most economical.

It is because of this acknowledged economy of the suspended arch in bridge construction, that I propose to draw a comparison between it and the upright ribbed arch, and to support with a few undisputed facts the proposition (based upon recent investigations of the compressive strength of cast steel) that an upright arched bridge for railway trains may be *more economically* constructed with cast steel by the method adopted in your Bridge, than is possible with the suspended arch, the length of spans in both cases being equal, and both being required to possess the same ability to resist deflection. This comparison will I think, vindicate my reasons for not wishing to adopt the suspension principle in your Bridge.

When iron or steel is subjected to any strain, either tensile or compressive, the material is lengthened or shortened in proportion to the force exerted. When released from the strain it resumes its original length, unless the force exerted exceeded its limit of elasticity. If this occurs, the material receives what is called a *permanent set*. In proportion as the piece under strain is lengthened, its diameter is reduced. When permanent set occurs, it fails

to recover its diameter as well as its length. Its ability to sustain a repetition or continuation of tensile strains, after loss of diameter occurs from permanent set, is proportionately lessened, and by frequently repeating the load that produced it, the material will be fractured. Sometimes a less one will, after a few repetitions, break the metal. When the piece is subjected to compressive strain, these conditions are reversed. The piece is shortened under the action of the force, and its diameter is increased. Permanent set, by increasing its diameter, increases to a certain extent its ability to resist the same force repeatedly applied.

If from any cause (for instance an error in workmanship), a member be subjected to a tensile strain beyond what it was designed to bear, such strain may exceed the limit of elasticity, and sooner or later it will snap asunder, and possibly cause the destruction of the Bridge. This was probably the reason why the Bollman truss (a rigid suspension bridge) gave way at Zanesville recently, under a much less load than it had been tested with, precipitating a locomotive and train into the river below. This could not occur with a member in compression, for if its length were so proportioned (as it should be) to its diameter as to insure crushing or permanently shortening without bending, the piece would be shortened by the extra strain, but would still perform its duty safely, as long as any other member of the structure.

We have no proof that iron or steel, when under compression, is any more liable to fracture by sudden jarring than when at rest; or that their strength is at all impaired by vibration when under compression. We do know, however, that they are more liable to fracture by sudden jarring or concussion when under tension, that their strength is impaired by continued vibration under tension, and that the liability to fracture by sudden jarring under tension increases with the increase of strain. The careful engineer, when using these materials in tension, will therefore leave a large margin within the elastic limit for safety; but for compressive strains, he may base his calculations on using them to the full limit of elasticity with entire safety. Cast iron is shown by Rodman to take a permanent set with 7,000 lbs. pressure, and many kinds with much less, and yet it is used in bridge construction by eminent engineers to 10,000, and even to 12,000 lbs. on the square inch in

compression; while wrought iron is rarely used in tension beyond 10,000 or 12,000 lbs., although it will stand from 18,000 to 25,000 lbs. before permanent set occurs.

Cast steel being much more homogeneous, and capable of being manufactured of any given quality with much greater certainty than cast iron, we are more fully justified in using it in compression up to the limit of its elasticity than we are in using cast iron.

By rolling and hammering cast steel, its strength is greatly increased. By tempering it in oil, Kirkaldy found its tensile strength increased 79 per cent. Hardening it increases the limit of elasticity of the material both for compressive and tensile strains, but also increases its liability to fracture by concussion when under tension. By being wire-drawn, it is possible to increase its ultimate tensile strength to 250,000 lbs. to the square inch. Fairbairn's recent experiments prove its ultimate strength in compression to be two and one-tenth times greater than in tension.

The limit of elasticity of mild rolled steel is about 50,000 lbs., but it is ordinarily used only to 20,000 lbs. by engineers for maximum tensile strains. By wire-drawing, it could probably be safely used to twice or thrice that strain. Hammered cast steel, designed for tools, when tempered, will sustain a crushing force of nearly 400,000 lbs., and its elastic limit will probably reach 120,000 lbs. It is possible that the steel prepared for stamping coins may considerably exceed that strength.

Experiments recently made at the Washington navy yard for me, through the courtesy and under the careful supervision of Chief Engineer Wm. H. Shock, U. S. N., Chief Engineer of the yard, show the elastic limit of sixty-eight samples of rolled cast steel under compression to average 55,008 pounds, and eight samples of rolled cast steel, each one inch in diameter and twelve inches long, to average 75,197 pounds per square inch. This steel was all of the ordinary tool steel of commerce, and made in Pittsburgh.

These facts show that the elastic limit of cast steel is greater in compression than in tension, and that this limit can be increased by proper manipulation of the material, for compression as well as for tension; and that we can use its full limit in compression, while we dare not go much beyond one-half of it in tension. How much the available difference is, in the present state of our knowledge of

the manufacture and treatment of the material, will only be shown by careful experiment to discover the steel best suited for each use. Enough is known, however, to assure us that the limit of elasticity is higher in compression than in tension, and if it were no greater, we cannot avail ourselves of more than 50 or 60 per cent. of this limit in tension, while the whole of it can safely be utilized in compression.

I will now proceed to examine whether there be any conditions involved in the use of the upright ribbed arch that will counterbalance the advantage to be gained by using the material in compression.

This investigation involves the inquiry, *firstly:* will the structure require more material by using the upright arch than the suspended one; and if so, how much more? And *secondly*, will its construction and erection be more costly?

Fig II

If the upright arch, A B, and the suspended one, C D, (Fig. 11) be each supported upon the points E F, and are each of the same form and weight, the compressive strain throughout the upright arch will be exactly equal to the tensile strain throughout the suspended one. The weight of the upright arch will exert no more power to force the points E F asunder than the weight of the suspended arch can exert to pull them together. Hence, the tensile strain borne by the metal of the suspended arch at C and D must be exactly equal to the compressive strain on that in the other at A and B. These facts being incontrovertible, it follows that if the upright arch be made of a material whose compressive strength is twice as great as its tensile strength, it will with equal safety sustain, with one-half of the material, as great a load as the suspended arch, so long as the form of its curve is preserved; because a bar of cast steel one inch square will safely sustain only 25,000 lbs. if

the weight be *suspended* by it, while it will safely sustain 50,000 lbs. if it be placed *under* the load in the manner of a post or column. If steel were but one-half stronger in compression than in tension, then two-thirds of the material only would be required to give the same strength in the upright arch that would be required for the suspended one. In that case, an upright arch having 1,000 tons of cast steel properly disposed throughout its length, would sustain as great a load as 1,500 tons in the suspended form. (To avoid complicating the explanation, we will consider the weight of the bridge itself, and the load it bears, together.)

I admit that the upright arch must be more thoroughly braced than the suspended one, but we should in this instance have 500 tons of material to spare for this purpose, between the abutments alone, for the steel does not cease at the towers in the suspended arch, but continues on over them, and usually half a span beyond them on each side, to the anchors; whilst with the upright arch it stops at the abutments.

The advocates of the suspension principle will assert that the influence of gravitation maintains the suspended arch in position against lateral movement, while the same influence would cause the other to topple over. This is, however, only true where but one rib of the arch or one cable is used, and only affects the argument where the span is greatly disproportioned to the width of the Bridge. The ingenious device to give lateral stability to suspension bridges will be equally effective in imparting it to the upright arched bridge of great span. The method adopted is to keep the points from which the cables are suspended widely separated, while the latter are drawn more closely together at their points of greatest depression. This is a reversal of the method proposed by Telford for his great cast-iron arch over the Thames, as the conditions in the suspended arch are exactly reversed. The ribs forming the upright arch, where the span is very great, would be placed widely apart at the abutments, and from thence would gradually incline toward each other until the summit of the bridge is reached. By spreading the arches at their bases, and fastening them to the abutments, we obtain even greater security against lateral flexure in the upright than in the suspended form, for each rib of the upright span springing from the masonry acts as a rigid brace, held at its lower extremity by the weight of the

abutment, and is competent to resist either tension or compression. In this case, resistance to lateral flexure would come from the weight of the abutments, while in the suspension bridge, it comes only from the gravity of the suspended mass — the resistance of the floors or horizontal bracing being the same in both cases.

The upright arch requires to be thoroughly braced to prevent a change of form under the moving load, as the flattening or straightening of any part of the curve in it tends to decrease its strength, whilst the curve of the suspended arch may be changed in a greater degree without endangering the structure. For railroad purposes, however, this could not be permitted in the suspended arch to any considerable extent.

In the upright arch, the parts lying between the centre and the abutments, called the haunches, mutually balance each other, and the imposition of a load on either haunch, if the arch be flexible, causes the other to rise, while the one sustaining the load is proportionately depressed. When this change of its form occurs, the centre of the arch moves horizontally towards the unloaded side; but if we prevent the centre of the arch from moving horizontally, the haunch cannot be depressed with the load equally distributed over it, except to the slight extent that is caused by the shortening of the steel under the compressive force exerted by the load. This movement is prevented by dividing the mass of the arch into an upper and lower member, and bracing these so effectually that their relative positions to each other cannot be altered under the severest strains to which they can be subjected. This gives stiffness to the haunches, and when one is loaded it is not only supported by its own rigidity, but by that of the unloaded one also, because the one cannot be bent downwards without bending the other upwards, and therefore the effect is really the same as though the load on one haunch was borne by two arches of but half the real span. This stiffness prevents the horizontal movement of the arch at the centre, and this is further prevented by securing the upper roadway of the Bridge to the crown of the arch, by which any horizontal movement of the arch would be transmitted through the roadway and resisted at the abutments.

The moving load affects the equilibrium of the suspended arch less than the upright one, but it nevertheless disturbs it to an extent that

requires a certain amount of bracing to prevent it from changing the form of its curve; and therefore all the bracing required for this purpose by the upright arch must not be counted against the latter in comparing the relative economy of the two systems.

Mr. Roebling says: "Wire cables, if guarded against oscillations, and not exposed to an undue tension, may be looked upon as of indefinite durability. Severe friction, short bending, constant vibration, high tension, and frequent shocks, will soon wear out the best material. The more we can reduce these exposures, the greater will be its durability." To preserve a suspended arch against ultimate destruction, it is therefore important to maintain its form against repeated alterations as far as possible.

The difference in the cost of erection of the arches would be considerably less for the suspended one than the other. The ribbed arch, however, can be erected without the use of false works beneath it, and is of all systems (except the suspension) the most cheaply erected. The ribbed arch with fixed ends may be self-sustaining for more than one-fourth of its length from each abutment, and the remainder can be put together by suspending it from cables supported on small temporary towers upon the abutments. It would be unnecessary to have these cables of a size sufficient to bear much more than the upper or lower member only of the rib beyond the point at which the arch would be self sustaining. If that were completed before the other one was put up, it would then aid in sustaining the other members required to finish the arch. The upright ribs would require to be braced from one to the other laterally, which would not be necessary if they were suspended. This however lessens the amount required in the roadways, while the lateral bracing required in the roadways of the suspension bridge cannot be much less.

The greater cost of the erection of the upright arch could not possibly equal the cost of the greater quantity of material required in the suspended one between the towers. As was before stated, however, the excess of material required for the arches or cables of the suspension bridge is not confined to that required in the arches between the towers, for these cables must be carried nearly or quite half the span beyond the towers on either side to the anchors, while the upright arch ceases at the abutments. This is more clearly shown in the accompanying wood cuts. The material required to

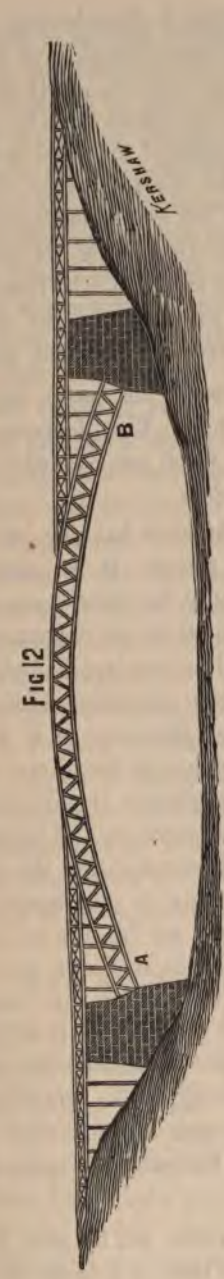


FIG 12

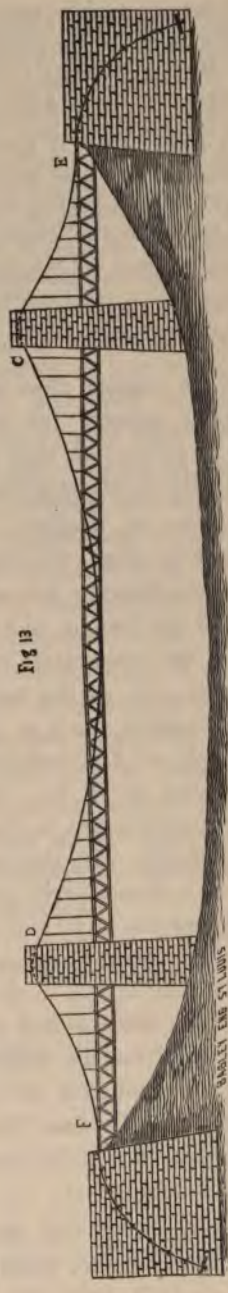


FIG 13

extend from one anchor, and over the towers to the other one, is usually double the length of the suspended arch, and it must be of equal cross section or diameter with that between the towers. If it be less than double the length, the portions beyond the towers must be greater in sectional area. Now, if the tensile strength of the steel be availiable to the extent of but half what it would be in compression, and double the length of material be required in the suspended arch to reach from anchor to anchor, it must follow, (excluding all question of cost of bracing the two systems,) that there will be four times as much material required from anchor to anchor in the suspension principle, as is needed in the upright arch between the abutments.

It will be claimed that the extension of the cables beyond the towers gives support for the approaches. This however utilizes but a small portion of their cost, for a much less expensive method of approaches is almost always practicable.

In the item of masonry, the upright arch has the advantage, as a glance at Figs. 12 and 13 will prove. If we assume that the two spans between the abutments and the towers are of equal weight, and the rise in one arch be equal to the deflection in the other, then the compressive force at A B is just equal to the tensile force at C D, and as the cables rest upon movable saddles on the towers, the tensile strain at C D will be the same at E F. This pulling strain at E F is therefore just equal to the thrust or compressive strain at A B. It will consequently require the same resistance to counteract it, that will be required at A B to resist the thrust of the arch, with this important difference—that the thrust of the arch is a *downward* pressure, tending to add weight to the masonry below the arch, and thus increase the friction of the mass (which is its element of resistance), while the strain of the cable is an *upward* one, and is in the direction to diminish this resistance. Assuming that the masonry required for the abutments be equal to what we require for the anchorage of the suspension bridge, then the masonry contained in the two towers supporting the cables will be that much more than the upright arch requires. Of course these propositions will be more or less favorable to either system in different localities.

If it were admitted that the material could not be used in compression with any higher value of strength than in tension, we would

still be able to provide the excess of bracing required in the upright arch, and the difference in the expense of erecting it, at less cost than is incurred by the other system, because of the necessity of carrying the cables beyond the towers; and we would in that case still effect a saving in the cost of masonry by using the upright arch.

In the cost of construction there can be but little difference. If an extraordinary high tensile strength be obtained in the steel to economize the quantity required, it would probably be used in the form of wire, and this would increase the cost of fabrication. In the form in which I propose to use it, three-fourths of the quantity required will be used almost as it comes from the makers, in bars about nine feet long, each weighing about 130 pounds, and three cents per pound extra will pay for finishing it ready to go into the structure.

I think all these facts clearly prove the economy of the upright ribbed arch over every other system of bridging with long spans.

The chief secret of the greater economy of iron suspension bridges rests in the fact that the limit of elasticity of iron wire is far greater in *tension* than the limit of elasticity for any form of iron known (except steel) when used in *compression*. Cast steel reverses this condition of things, and the adoption of it in bridge construction must result in reversing this advantage of economy and giving it in favor of the upright arch, and thus enable us to make spans of equal if not greater length, and with as great safety, as by the suspension principle. Mr. Roebling asserts that suspension bridges may be safely made of iron wire of half a mile span, and that when steel is used in their construction, nearly double this span may be accomplished with equal security.

MANNER OF USING THE STEEL.

To obtain the highest value of cast steel in compression, with the greatest economy in construction, I think it should be used in the tubular form. Although cast steel tubes have been recently drawn cold by hydrostatic pressure in France, from steel expressly prepared for the purpose, I can not learn that the process has been carried to any extent beyond the production of gun barrels. It is quite possible that this method may in the future, furnish steel much superior for bridging to anything we can now obtain. I have but little doubt that methods of manufacturing and tempering steel in this form,

for bridge construction, will soon be discovered, by which a much higher value of strength may be safely used in construction. As the use of cast steel in bridge building is comparatively in its infancy, I have deemed it proper to use the material at a much safer limit as regards its ultimate strength than my judgment would otherwise dictate. I feel assured that the structure would be entirely safe to bear a far greater load that can be placed upon it, if its arches contained but one-half of the steel that will form them. When this material comes to be universally used in bridge construction in the place of wrought and cast iron, as it inevitably will be because of its greater economy, the very large margin for safety provided by the liberal use of this material in your Bridge will be more fully appreciated.

To insure a uniform quality and high grade of steel at the lowest prices, and at the same time avail myself of the advantages of the tubular form of construction, I propose to have the steel rolled for the arches in bars of nine feet length, and of such form that ten of them shall fill the circumference of a nine-inch lap-welded tube inch thick, in the manner that the staves of a barrel fill the hoops. This would virtually form a steel tube nine inches in diameter and of six inches bore, the steel being about $1\frac{1}{2}$ inches thick, and would be much less expensive than if the tube were rolled or drawn in one piece. The manufacture of the steel in such small bars will insure a more uniform quality in the metal, and in the tube each bar will be supported against deflection in every direction. The tubes will be retained in their positions by an effective system of bracing, which will sustain the *voussoirs*, or pieces against which the tubes are butted throughout the arch. The upper and lower members of each arch will each be formed of two courses of these tubes, from end to end of the arch, each tube having a sectional area of 36 square inches at the summit of the arch. As each span would be made up of four arches, and each arch of four of these tubes, the span would have an aggregate sectional area at that part of 576 cubic inches of steel. The tubes, for about 20 feet of their lengths nearest the abutments, would require one-half more sectional area to resist the greater strains at those points. The tubing in which the steel bars would be inclosed would effectually protect the latter from the weather.

I am gratified in being able to state that proposals from several of our leading steel makers in the United States have been received,

and also from some of the most celebrated in Europe, among whom I may name Naylor & Co., of England; Petin, Gaudet & Co., of France; and Fred. Krupp, of Prussia; all offering to furnish the steel, and agreeing to guarantee its strength fully up to the standard required.

The importance of being guided by the very best lights that can be obtained from practical and careful experiment, and the great interests involved in the safety and permanency of the structure, fully convinced me, at the inception of this enterprise, of the necessity of instituting a careful series of experimental tests of the materials to be used, and also determined me to have every part of the structure thoroughly tested to a degree of strain much beyond what it can by any possibility be subjected to when in the Bridge.

For this purpose, I am having a powerful machine made, that will be capable of carefully testing every member used in its construction.

ESTIMATE OF COST OF THE BRIDGE.

I. Superstructure of Bridge.

One Arch of 515 feet span.

848.05	tons of cast steel, in ribs, at $320	$271,376 00
158.12	" wrought iron, in ribs. at $200	31,624 00
373.99	" rolled iron, in ribs, at $140	52,358 60
60.56	" cast iron, in bed-plates, at $80	4,844 80
196,000	feet, b. m., pine timber and plank, at $50 per M	9,800 00
23,000	" oak " " at $70 per M	1,610 00
191	squares Nicholson pavement, at $20*	3,820 00
34	tons of spikes, bolts, etc., at $120	4,080 00
7	" cast iron, in roadways, at $80	560 00
26,500	square feet lining, of galvanized iron, at 20c	5,300 00
1,060	linear feet of railing, at $6	6,360 00
1,060	" cornice, at $1 50	1,590 00
Painting		12,000 00
Raising		20,000 00
Testing		12,000 00
Engineering and contingencies, 10 % **		43,732 34
Total		$481,055 74

Or $934.09 per linear foot.

* The cost of the plank floor, on which the blocks of the pavement rest, is estimated separately.

** The contingencies have been estimated at only 10 per cent. in this case, because, as before stated, the prices for material and workmanship in the estimate are based on actual proposals received from reliable parties.

Two Arches of 497 feet span.

The cost of these two spans was calculated from the same data as the span of 515 feet, and was found, including 10 per cent. or $84,983 36 for contingencies, to amount to	$934,819 16

Anchor Rods between Shorbacks.

772 tons of cast steel, at $34	$30,240 00
82 " cast iron (plates) at $30	2,560 00
Contingencies, $10\frac{0}{0}$	3,280 00
Total	$36,080 00

Superstructure on Piers and Abutments.

16.4 tons of rolled iron, in beams, at $120	$1,968 00
35,000 feet, b. m., pine timber, at $50 per M	1,750 00
9,000 " oak timber, at $70 per M	630 00
50 squares Nicholson pavement, at $20	1,000 00
391 linear feet iron railing, at $6	2,346 00
Contingencies, $10\frac{0}{0}$	769 40
Total	$8,463 40

Summary of Cost of Superstructure.

One span of 515 feet	$481,055 74
Two spans of 497 feet each	934,819 16
Anchor rods	36,080 00
Superstructure on piers and abutments	8,463 40
Total cost of superstructure	$1,460,418 30

II. Substructure.*

Caissons, piles, and coffer-dam		$132,700 00
Sinking caissons and machinery		65,000 00
Dredging and pumping		87,600 00
Coffer-dam for west abutment		18,000 00
5,178 cubic yards concrete, at $8		41,424 00
55,318 "	limestone masonry, at $14	774,452 00
3,784 "	sandstone masonry, at $15	56,760 00
5,109 "	granite masonry, at $32	163,264 00
Engineering and contingencies, 15 per cent.		200,880 00
Total		$1,540,080 00

* The masonry is all under contract, at prices below those introduced into the estimate.

III. Approaches.

12,000	cubic yards earth excavation, at 35c	4,200 00
200	" rock excavation, at $1 50	300 00
3,292	" limestone masonry, in approaches over levee, at $14	46,088 00
8,237	cubic yards sandstone masonry, at $15	123,555 00
1,244	" granite masonry, at $32	39,808 00
133	" cut sheeting, at $16	2,128 00
793	" coursed rubble, at $9	7,137 00
4,738	" rubble masonry, at $7	33,166 00
518	" brick arch, at $12	6,216 00
2,907	" brick masonry, at $10	29,070 00
559	" concrete, at $7	3,913 00
20.7	" dimension stone, at $20	414 00
3,802	" broken stone ballast, at $2	7,604 00
1,440	linear feet piling, at $1	1,440 00
121.95	tons cast iron, at $80	9,756 00
307.5	" wrought and rolled iron, at $140	43,050 00
1,188	linear feet railing, at $6	7,128 00
1,200	" " in trestle, at $3	3,600 00
487	" upper roadway (including lining, etc., and Nicholson pavement), at $40	19,480 00
816	linear feet upper roadway (including lining), at $31	25,296 00
518	linear feet lower roadway, at $22	11,396 00
371,458	feet, b. m., pine timber and plank, at $50 per M	18,572 90
68,150	" oak plank, at $70 per M	4,770 50
Stairs, etc		10,000 00
Station on Third street and entrance		15,000 00
Contingencies, 10 per cent		47,308 84
	Total	$520,397 24

IV. Tunnel.*

118,460	cubic yards earth excavation, at 35c	41,461 00
14,400	" refilling, at 20c	2,880 00
4,272	" concrete, at $7	29,904 00
16,493	" brick arch, at $12	197,916 00
2,380	" coursed rubble, at $9	21,420 00
58	" coping, at $40	2,320 00
4,024	" ballast, at $1 50	6,036 00
Sewers		35,000 00
Land damages, ventilators, etc		20,000 00
Contingencies and engineering, 15 per cent		53,540 55
	Total	$410,477 55

* The tunnel is 4,800 feet in length; the open cut south of Clark avenue 700 feet.

Assuming that if the Bridge had been finished, and one-half of this quantity (5 millions) had been brought over in railroad cars, and the other half only by teams, we would have, at 80 lbs. to the bushel, 200,000 tons; and for the other half, at 115 bushels per team, 43,477 teams, making the total of coal crossing the Bridge, 200,000 tons in bulk, and 48,694 teams.

RAILROAD AND FOOT PASSENGERS.

The railroad passengers in 1866 (figures of 1867 could not be obtained) coming to and from St. Louis, were 479,200. It is safe to estimate that 2,000 foot passengers would have crossed the Bridge per day, making per year, 700,000.

FARMERS' TEAMS.

There were brought by wagons across the river in the year 1867:

Wheat	355,320	bushels.	20,000 teams.
Barley	10,000	"	350 "
Corn	72,650	"	6,000 "
Oats	10,000	"	350 "
Rye	7,600	"	300 "
Flour	127,771	barrels.	15,593 "
			42,593 "

Allowing an average of only 100 farmers' wagons per day crossing and recrossing with vegetables, fruits, etc., we can add 36,500 teams, making a total of 79,093 teams.

HORSE RAILROADS AND PLEASURE CARRIAGES.

A double-track horse railroad, through the upper roadway of the Bridge, carrying an average of 1,000 passengers daily, passing through the principal portion of East St. Louis and connecting with the numerous lines centering at Third street and Washington avenue, in this city, will be in constant operation. Of pleasure carriages, hotel, express, and baggage wagons, an estimate of one hundred per day is much below the present intercourse between the two shores.

CATTLE.

There were imported and exported to and from St. Louis in 1867, by three railroads alone, as follows:

RAILROAD FREIGHT AND TONNAGE.

The freight forwarded to and from St. Louis during the year 1867 by three different railroads which have their termini at East St. Louis is as follows:

	Tons.
St. Louis, Alton and Terre Haute	484,685
Ohio and Mississippi	333,175
Chicago, Alton and St. Louis	249,550
Total Tonnage	1,067,410
From which deduct *Coal*	300,000
Total Merchandise	767,410

COAL.

The quantity of coal brought to East St. Louis by rail does not all cross the river. A large portion is loaded into barges, and by them discharged directly into steamers lying on the Levee, and another portion is transferred by barges to this side of the river, from whence it is loaded into wagons and distributed among the factories and through the city generally. The largest portion is, however, brought over directly by wagons.

The Illinois and St. Louis Railroad and Coal Company delivers about 250,000 bushels of coal per month on the eastern side of the river, of which 150,000 are sold directly to steamboats, and 100,000 are transferred in barges to the city, and from these it is loaded into wagons. The first item, being a direct transfer to steamboats, does not enter into our calculations, but it is fair to presume that of the 100,000 bushels per month which are transferred in barges, and have to be loaded into wagons and pulled up the steep Levee, one-half at least would be loaded into wagons directly from the cars, and cross the smooth, level Bridge with great saving to the teams, as well as in wear and tear of wagons.

Assuming, therefore, that of the whole 250,000 bushels per month, only 50,000 will cross the Bridge, we have 600,000 bushels per annum crossing by wagon, at 115 bushels to a four horse wagon, making 5,217 teams.

One of our largest dealers and owners of coal mines delivers seven and a-half millions of bushels in this city, and the other large dealers combined, about two and a-half millions, making ten millions of bushels; all of which cross by wagon.

4. Each foot passenger on the Ferry now pays 5 cents. On the Bridge the charge will be 3 cents, and the delays incident to the ferry will be avoided.

5. Farmers' wagons are now charged 50 cents. The Bridge's charge will be 40 cents.

6. The charge for cattle is now 12½ cents; for hogs and sheep, 4 cents. The proposed charges are 10 cents and 3 cents respectively.

ANNUAL SAVING TO COMMERCE.

Had the commerce of St. Louis, in 1867, passed over the Bridge, the following amounts would have been saved to our merchants and the community:

Sixty Cents difference in transporting 767,400 tons merchandise	$460,440
Reduction of 10 per cent in the price of coal, on ten million bushels, at 15 cents	150,000
The loss to Commerce through total interruption on account of ice is certainly not less than	100,000
The cheapening of fruit, garden vegetables, and feed	50,000
480,000 Railroad passengers at 25 cents	120,000
Making a total of	$880,440

The above amount would have been saved to the community in 1867, if a bridge had been in operation at that time. In 1871 when it will be finished, the saving to our citizens will amount to millions.

This estimate of revenue being based on statistical figures, official data and the most reliable information, without recourse to fantastic or high-wrought ideas, or imaginary profits, and giving a plain statement of the commerce and travel across the Mississippi opposite our city during the year 1867, brings us to the consideration of the

PROSPECTIVE REVENUE

of the Bridge after its completion in 1870 or 1871. It is barely possible for the mind to keep within cool, calculating bounds when contemplating the future of St. Louis, with her unbounded resources and unrivaled situation. Located in the centre of a valley of exhaustless fertility, on a river which traverses the continent, and on which hundreds of steamers are constantly engaged in pouring the treasures of that valley into her lap, she is now extending her railroads westward, northward, and southward, and will, by the time the Bridge is completed, become the recipient of

countless stores of metals, coal, lumber, cereals, hemp, tobacco, and cattle, the products of the favored regions which her extended railroads are approaching.

The Southwest Pacific and Iron Mountain Railroads are now in the hands of capitalists with ample means, and will in the course of two years be finished, the one through the fertile prairies of Southwest Missouri to the rich lead mines of Granby, and on to the Gulf of Mexico; the other through the Iron Mountain, Pilot Knob, Washington, and St. Francis mines, to connect with the Southern system of railroads opposite Columbus and Memphis. The North Missouri, now within thirty miles of the State line, will soon enjoy an unbroken railway communication with St. Paul. Its West Branch will also connect with the Union Pacific Railroad, Eastern Division, within the month of November next, and perhaps in twelve months more, with the Union Pacific, at Omaha; this with the Missouri Pacific, already connected with the Union Pacific, Eastern Division, secures to St. Louis the two great trunk lines which run to the Pacific Ocean. The point of intersection of these transcontinental railways, with a majestic river flowing from the frozen regions of the North almost to the Tropics, may be compared to the grand cross-roads of the American Continent. This intersection is made almost at the centre of the most magnificent system of inland navigation in the world, and almost in the very heart (whether considered geographically, or with reference to the extremes of climate) of the largest and most fertile valley on the earth. At this point St. Louis is located, and her future can scarcely be over-estimated.

The construction of this Bridge will stimulate the production and fabrication of iron, encourage the erection of manufactories of all kinds, and invigorate the whole industrial and commercial life of the city. It will make the thriving city of East St. Louis, a large and populous manufacturing place, where the artisans of this city can find cheap homes, and by which the intercourse between the two, amalgamated in interest, will be kept in constant activity. The consumption of coal and its transport across the Bridge will be enormous, and no more embargoes by ice, which sometimes isolate us from the East, render our ferries useless, and raise the price of fuel five-fold, will be suffered to palsy commerce and entail distress upon the poor.

Taking all these facts into consideration, it is safe to assume that at the time when this Bridge will be finished, and the com-

pleted railroads will pour all their wealth of mineral and produce into our lap, to be transported East across the Bridge, while the East again will send the products of her industrial establishments and factories westward across the same highway—when peace and prosperity will add a fresh impulse to enterprise and commerce—that the ratio of increase in 1871 will be at least 50 per cent. over that of 1867. Nothing invites commerce so much as facilities of transportation; nothing invites population so much as facilities of communication. The receipts and revenues of the ferry companies afford not the remotest criterion by which to calculate the prospective revenues and receipts of such a Bridge. They can only be depended upon as establishing a minimum estimate, for we might as well calculate the prospective receipts of a railroad by taking as a basis, the tolls of the country turnpike along which it passes.

Counting, therefore, 50 per cent. increase as a perfectly safe basis, the receipts of the Bridge would be $1,136,260 in 1871. Calculating the receipts of the Bridge, however, when first opened to be only one million of dollars, it will show the following financial results:

Total cost of Bridge, Land, and Approaches, (estimated cost being $4,486,953,) say	$4,500,000
Interest at 6 per cent., *gold*, for two years on half the sum	270,000
Premium on gold 40 per cent	108,000
Total cost	$4,878,000

RESOURCES.

4,000 bonds of 1,000 each, bearing 6 per gold cent. interest, at "$950,"(5 per cent. discount)	$3,800,000
Full paid stock	1,000,000
Remnants of valuable strips of land available after condemnation*	150,000
	$4,950,000

* This amount is much below what will likely be realized from the sale or rent of condemned property, which will remain in strips, unavailable for bridge purposes, but of great prospective value.

REVENUE.

		1867.	1871.
From all sources as stated above		$757,507	$1,000,000
Less interest	$270,000		
" premium on gold	108,000		
" 1 per cent. sinking fund †	40,000		
" office salaries	20,000		
" repairs	20,000		
" taxes, police, etc	10,000		
		$468,000.	$468,000.
Leaving a surplus on estimate of 1867		$289,507.	
" " " 1871			$532,000

It will be readily seen that, resting even on the incredible assumption that the receipts of the Bridge will not exceed one million of dollars for the next thirty years, the bonds will all be paid, and the Bridge be free of debt at the expiration of that time. In the estimate of expenditures, the item of 40 per cent. premium on gold is included, amounting to $96,000 annually, which is to diminish gradually and with certainty, and will cease entirely when specie payment is resumed.

Whatever shape the financial policy of the Company may assume, I feel safe in stating that rarely has an enterprise been inaugurated which appeals so strongly to the support of our citizens of all classes, which promises so much to add to the welfare and prosperity of the city, and which offers such a safe and remunerative return for the labor and capital invested in it.

CONCLUSION.

The contract for the entire masonry of the Bridge has been awarded to Mr. James Andrews, of Allegheny City. This gentleman has had great experience in similar undertakings, and enjoys a high reputation for skillfulness, energy, and faithfulness in the prosecution of his contracts. The masonry thus far executed has been constructed by him under this contract.

I am under many obligations to Mr. Wm. H. Shock, U. S. N., Chief Engineer of the Navy Yard at Washington, for the very careful and interesting series of experiments with cast steel made by him for this work at my request. Those experiments were made with the testing machine used by Col. Rodman, U. S. A., in

† An annual sinking fund of $40,000 at compound 6 per cent. interest, will realize in thirty years $4,376,316, sufficient to pay off the whole bonded debt.

his well known experiments on cast iron for cannon. The results obtained by Mr. Shock have been before alluded to in this report, and are very valuable as defining the limit of elasticity in the various samples of crucible cast steel tested by him, thus giving assurance that by using the steel in the manner I propose doing, we are far within the limits of safety.

The work in the drawing room has been chiefly executed by Mr. Wm. Rehberg, C. E., and the remarkable neatness and precision observed by him in this important part of the work, has relieved me of much care, and entitles him to the kind consideration of the company.

The first stone in the construction of the Bridge was laid upon the bed rock of the river, within the coffer dam inclosing the site of the western abutment, on the 25th of February, 1868, in the presence of the President, Directors, and several of the Stockholders of the Company, and of the Mayor, Hon. Jas. S. Thomas, and many other prominent citizens of St. Louis.

I cannot close this report without availing myself of the opportunity it affords of expressing my grateful appreciation of the unwavering confidence and support I have received from the Directors and Stockholders of the Bridge Company, from its inception to the present time, during which period the plans adopted by you have been subjected to a degree of public and private criticism and misrepresentation rarely attending such enterprises.

Respectfully submitted.

Your obedient servant,

JAMES. B. EADS.

ENGINEER'S OFFICE ILLS. & ST. LOUIS BRIDGE CO.,
June 1st, 1868.

CERTIFICATE OF CHANCELLOR W. CHAUVENET, LL.D.,

Affirming the correctness of the Scientific Investigations and Computations made for the Bridge.

WASHINGTON UNIVERSITY,
ST. LOUIS, May 2, 1868.

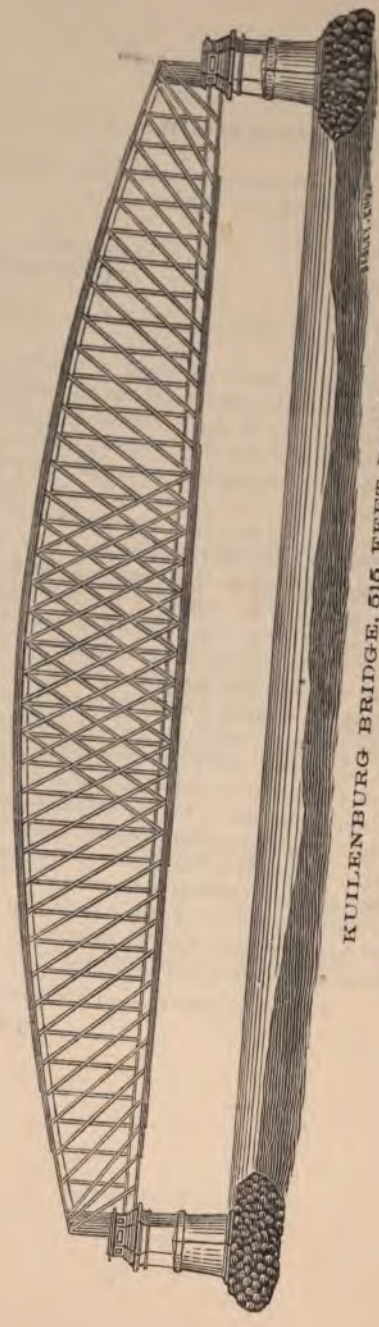
JAMES B. EADS, *Chief Engineer Illinois and St. Louis Bridge:*

My Dear Sir—In compliance with your request, I have carefully examined the theoretical investigations and the computations made in your office for the construction of the St. Louis Bridge. These investigations relate to the stability and safety not only of the steel arches, but also of the piers and abutments by which they are supported. They are based upon established principles, and are entirely correct. They prove not only that the arches will support themselves and any load that will ever be placed upon them, but also that they will withstand the additional strains that can be produced by the greatest supposable increase of temperature (causing an expansion of the steel), or by the action of the wind in the most violent storms. The piers and abutments are proved to be more than sufficient to sustain, 1st, the arches themselves, with their maximum loads; 2d, the greatest supposable pressure of the water in the highest stage of the river and with the swiftest current; 3d, the additional force resulting from the action of wind upon the whole superstructure; 4th, the greatest possible pressure exerted by accumulated ice. In short, every question involving in any degree the stability and safety of the Bridge in any of its parts, is examined, not merely by means of rough approximations, but by the most precise application of mathematical principles which the nature of the case will permit.

As I have not only examined into the correctness of the principles involved, but have also verified all the numerical computations, I take pleasure in expressing to you my entire confidence in the correctness of your conclusions. The Bridge erected in accordance with them will be a new triumph of the science of engineering, and an honor to our city.

Very respectfully, your obedient servant,

WM. CHAUVENET.



KUILENBURG BRIDGE, 515 FEET LONG.

APPENDIX.

MATHEMATICAL

INVESTIGATIONS AND COMPUTATIONS

FOR THE CONSTRUCTION OF THE

ILLINOIS AND ST. LOUIS

BRIDGE.

Mo. Republican Print.

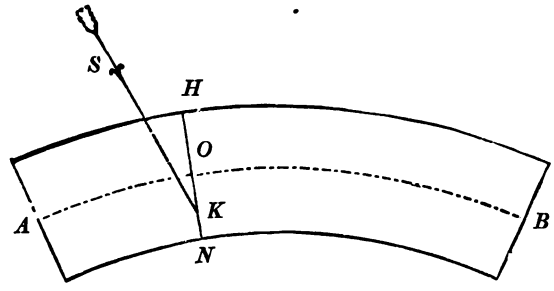
of a free arch. Also in the central parts of the arch the strains were found to be more uniform, and for partial loads corresponded more nearly to the case of a load uniformly distributed over the whole arch, though they were greater near the ends; indicating that the sectional areas of the ribs for the greater part of the span could be reduced, if for a short distance near each pier they were increased, thus effecting an important saving of material. This was true notwithstanding the fact that changes of temperature were found to produce much greater strains in the fixed than in the free arch. These results led to a third computation

C. In this computation the arch was treated as circular, the ends were supposed fixed, and the ribs were assumed to be strengthened at the ends by increasing their sectional area for a distance from each pier equal to one-twelfth of the span. In this form, it was found, as was anticipated, that the deflections were somewhat further reduced, the conclusions drawn from the second computation were confirmed, and the advantage of increasing the strength of the end pieces was fully established.

We propose to give here the details of the last computation only, as the results thus obtained are adopted as definitely establishing the main features of the proposed bridge.

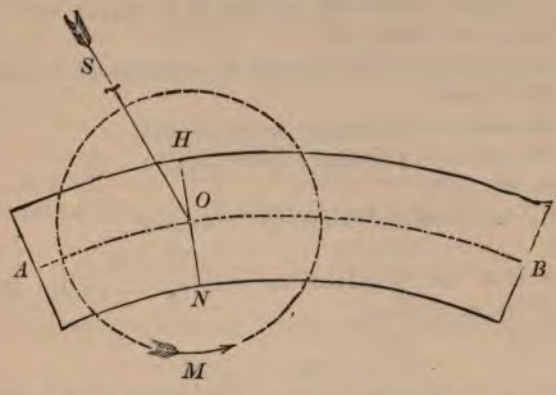
THE FIXED ARCH WITH STRENGTHENED END PIECES.

Fig. 1.



If AOB (Fig. 1) is the centre line of an elastic arch any normal section of which as HN, is a symmetrical figure with respect to the centre line, and if different external forces are acting in the plane of the centre line, then the internal forces acting in the plane HN on any portion AO of the arch have to be in equilibrium with all the external forces acting on the same portion. These external forces may be supposed to be first reduced to their resultant, a single force S acting at some point K in the normal plane.

Fig. 2.



The force S acting at K may then be replaced by an equal force S (Fig. 2) acting at the centre O of the normal section, and a couple of forces whose moment M tends to produce rotation about the point O.

Finally, the force S may be replaced by two components S' and S'', one acting at O perpendicular to the normal plane, that is, in the direction of the arch at this point; the other acting in the normal plane, that is, in the direction of the normal to the arch at O. The force S' produces a *compressive* or *tensile strain* in the direction of the arch; the force S'', the *shearing strain*. These strains are supposed to have no tendency to produce a change of figure of the arch; the strain S' not exceeding the limit of elasticity of the material, a proper sectional area of the rib being furnished, and producing only a small variation in the length of the centre line which at present we neglect*; while the shearing strain S'' is resisted by the cohesion of the material in the normal plane and by a suitable system of braces.

Omitting for the present the strains S' and S'', let us consider first the effect of the moment M at the several points of the arch.

The moment M produces strains of the same absolute values above and below the neutral axis, compressive on one side and tensile on the other side of this axis. There results a deflection of the centre line, and hence M is called the *moment of flexure*.

If the vertical thickness of the arch is small compared with the radius of the arch, and if the strains do not exceed the limit of elasticity, experience shows that a plane section HN normal to the curve of the arch before flexure remains normal to the curve of the deflected arch.

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* The influence of the compression in reducing the length of the arch will be considered in the course of the numerical calculations hereafter.

If we put

φ = the angle which the normal at O makes with a vertical line *before* flexure,

φ_1 = the angle which the normal at O makes with a vertical line *after* flexure,

R = the radius of curvature *before* flexure,

ρ = the radius of curvature *after* flexure,

s = the arc AO,

ds = the infinitesimal element of the arc s,

the condition that the length of the centre line, or arc AO, is not changed by the flexure, gives

$$ds = -Rd\varphi = -\rho d\varphi_1. \tag{1}$$

The arch being conceived to be composed of fibres of infinitesimal thickness lying in the direction of the curve, the position of any fibre may be expressed by coördinates u and v, the axis of u being a horizontal line through O perpendicular to the vertical plane of the arch, and the axis of v being the normal line at O, or intersection of the normal and vertical planes at this point. We shall assume that the axis of u is the *neutral axis.** The condition that the normal section HN remains normal to the curve AO at O, during flexure, requires that fibres at different distances from the neutral axis should be compressed or elongated by quantities proportional to those distances, that is, proportional to the values of v. Conceiving therefore two consecutive normal planes to be drawn, the length of the infinitesimal portion or element of any fibre intercepted between these two planes will be, before flexure $ds + vd\varphi$, and after flexure $ds + vd\varphi_1$; the element has therefore been increased in length by $(d\varphi_1 - d\varphi)v$. If then

ε = the modulus of elasticity of the material composing the arch,

$dudv$ will be the area of the section of the fibre, and $\varepsilon . dudv$ will be the force necessary to elongate the element $ds + vd\varphi$ by its own length; and hence the force necessary to elongate it by the quantity $(d\varphi_1 - d\varphi)v$ will be

$$\frac{(d\varphi_1 - d\varphi)v}{ds + vd\varphi} . \varepsilon . dudv,$$

* For a straight prismatic beam this assumption is exact, and it is sufficiently correct for a curved beam whose radius is very great in comparison with its thickness.

or substituting $Rd\varphi$ for ds,

$$\frac{d\varphi_1 - d\varphi}{d\varphi} \cdot \varepsilon \cdot \frac{dudv.\ v}{R+v},$$

and the moment of this force about the neutral axis is the force multiplied by v, i. e.,

$$\frac{d\varphi_1 - d\varphi}{d\varphi} \cdot \varepsilon \cdot \frac{dudv.\ v^2}{R+v}.$$

The total moment M of all the fibres is therefore

$$M = \frac{d\varphi_1 - d\varphi}{d\varphi} \cdot \varepsilon \cdot \iint \frac{dudv.\ v^2}{R+v},$$

the limits of the integrals being determined by the form of the normal section.

The figure of the normal section being symmetrical with respect to the neutral axis, the values of $\frac{dudv.\ v^2}{R+v}$ at equal distances above and below this axis will be

$$\frac{dudv.\ v^2}{R+v} \text{ and } \frac{dudv.\ v^2}{R-v},$$

whose sum is

$$\frac{2}{R} \cdot \frac{dudv.\ v^2}{1 - \frac{v^2}{R^2}};$$

and since in our arch $R = 673.5$ feet and the mean value of $v = 4$ feet, the mean value of the fraction $\frac{v^2}{R^2}$ is $\frac{1}{28350}$; hence in taking the integral no appreciable error is committed by omitting this fraction, and we may take, according to the usual practice,

$$M = \frac{d\varphi_1 - d\varphi}{d\varphi} \cdot \frac{\varepsilon}{R} \cdot \iint dudv.\ v^2.$$

The integral $\iint dudv.\ v^2$ is simply the *moment of inertia* of the normal section. Where the arch is composed, as in the present case, of two members of small sectional areas at a considerable distance from the neutral axis, the moment of inertia may be computed with sufficient exactness by conceiving all the internal forces of each member to act only at its centre of gravity; so that, if

ω = the area of a section of a member,
δ = the distance of centres of gravity of the members,
θ = the moment of inertia of the normal section of the arch,

$$(6)\quad \begin{cases} Q_1 = Q, \\ P_1 = qna - P, \\ N_1 a = -Na + 2Pa - qna\left(2a - \dfrac{na}{2}\right), \\ \qquad = -Na + 2Pa - 2qna^2\left(1 - \dfrac{n}{4}\right), \end{cases}$$

by which the resistances at the right pier can be determined if those at the left pier are known. In deducing the last equation, or the equation of the moments, we have taken the moments about A_1; the same equation can be found by taking the moments about A, observing that from the nature of the couples denoted by Na and $N_1 a$, they may be transferred to any point in the system under consideration, and hence, the whole arch being here considered, they may both be regarded as acting either at A_1 or at A.

If now, δ being the distance of the members, we put

ω = the sectional area of the centre piece, from G to G_1,

ω_1 = the sectional area of the end pieces, from A *to* G, and from A_1 to G_1,

$\theta = \dfrac{\omega\delta^2}{2}$ = the moment of inertia of the section of the centre piece,

$\theta_1 = \dfrac{\omega_1\delta^2}{2}$ = the moment of inertia of the section of an end piece,

we shall have, by equation (3), for the centre piece from $x = pa$ to $x = (2-p)\,a$, that is, from $\varphi = \gamma$ to $\varphi = -\gamma$,

$$(\text{VII})\qquad d\varphi_1 - d\varphi = \frac{RM}{\varepsilon\theta}\,d\varphi,$$

and for an end piece from $x = 0$ to $x = pa$ and from $x = (2-p)\,a$ to $x = 2a$; that is, from $\varphi = \alpha$ to $\varphi = \gamma$, and from $\varphi = -\gamma$ to $\varphi = -\alpha$,

$$(7)\qquad d\varphi_1 - d\varphi = \frac{RM}{\varepsilon\theta_1}\,d\varphi.$$

These equations result from a consideration of the internal forces, and, as these are in equilibrium with the external forces, we have only to substitute in them the values of M in terms of the external forces P, Q, Na and qna.

Considering now the forces acting on the portion AF of the arch, we have

(a) For loaded part, from $x = 0$ to $x = na$,

$$M = Na + \frac{qx^2}{2} - Px + Qy;$$

or by (4),

(8 a) $$M = NR\sin\alpha - PR(\sin\alpha - \sin\varphi) + QR(\cos\varphi - \cos\alpha) + \frac{qR^2}{2}(\sin\alpha - \sin\varphi)^2.$$

(b) For unloaded part, from $x = na$ to $x = 2a$,

$$M = Na - Px + Qy + qna\left(x - \frac{na}{2}\right),$$

or by (4),

(8 b) $$M = NR\sin\alpha - PR(\sin\alpha - \sin\varphi) + QR(\cos\varphi - \cos\alpha) - qR^2\sin\varphi(\sin\alpha - \sin\beta) + \frac{qR^2}{2}(\sin^2\alpha - \sin^2\beta).$$

Supposing now β to fall between $+\gamma$ and $-\gamma$, we obtain from (VII) and (7), by substituting the values of M from (8 a) and (8 b) and integrating,*

(a) For loaded part:

END PIECE.

(9a) $$\varphi_1 - \varphi = \frac{R^2}{\varepsilon\theta_1}\left\{\left(N\sin\alpha - P\sin\alpha - Q\cos\alpha + \frac{qR}{2}\sin^2\alpha + \frac{qR}{4}\right)\varphi - (P - q\alpha)\cos\varphi + Q\sin\varphi - \frac{qR}{4}\sin\varphi\cos\varphi + A\right\},$$

CENTRE PIECE.

(IX a) $$\varphi_1 - \varphi = \frac{R^2}{\varepsilon\theta}\left\{\left(N\sin\alpha - P\sin\alpha - Q\cos\alpha + \frac{qR}{2}\sin^2\alpha + \frac{qR}{4}\right)\varphi - (P - q\alpha)\cos\varphi + Q\sin\varphi - \frac{qR}{4}\sin\varphi\cos\varphi + A_1\right\}.$$

(b) For unloaded part:

CENTRE PIECE.

(IX b) $$\varphi_1 - \varphi = \frac{R^2}{\varepsilon\theta}\left\{\left(N\sin\alpha - P\sin\alpha - Q\cos\alpha + \frac{qR}{2}\sin^2\alpha - \frac{qR}{2}\sin^2\beta\right)\varphi - (P - q\alpha + qR\sin\beta)\cos\varphi + Q\sin\varphi + B_1\right\},$$

END PIECE.

(9 b) $$\varphi_1 - \varphi = \frac{R^2}{\varepsilon\theta_1}\left\{\left(N\sin\alpha - P\sin\alpha - Q\cos\alpha + \frac{qR}{2}\sin^2\alpha - \frac{qR}{2}\sin^2\beta\right)\varphi - (P - q\alpha + qR\sin\beta)\cos\varphi + Q\sin\varphi + B\right\},$$

---

* Employing the known integrals $\int d\phi.\sin\phi = -\cos\phi$, $\int d\phi.\cos\phi = \sin\phi$, $\int d\phi\sin^2\phi = \frac{1}{2}(\phi - \sin\phi\cos\phi)$.

in which A, B, A_1, B_1, are constants of integration. To determine relations between these constants, whence finally the values of the constants themselves will be found, we observe that (9 a) and (IX a) must be identical for $\varphi = \gamma$; and (9 b) and (IX b) must be identical for $\varphi = -\gamma$; hence

$$0 = (\theta_1 - \theta)\left\{\left(N\sin\alpha - P\sin\alpha - Q\cos\alpha + \frac{qR}{2}\sin^2\alpha + \frac{qR}{4}\right)\gamma - (P - q\alpha)\cos\gamma + Q\sin\gamma - \frac{qR}{4}\sin\gamma\cos\gamma\right\} + \theta_1 A_1 - \theta A,$$

$$0 = (\theta_1 - \theta)\left\{\left(-N\sin\alpha + P\sin\alpha + Q\cos\alpha - \frac{qR}{2}\sin^2\alpha + \frac{qR}{2}\sin^2\beta\right)\gamma - (P - q\alpha + qR\sin\beta)\cos\gamma - Q\sin\gamma\right\} + \theta_1 B_1 - \theta B;$$

and from those equations, by subtraction,

(10)

$$0 = (\theta_1 - \theta)\left\{2\gamma\left(N\sin\alpha - P\sin\alpha - Q\cos\alpha + \frac{qR}{2}\sin^2\alpha + \frac{qR}{8} - \frac{qR}{4}\sin^2\beta\right) + qR\sin\beta\cos\gamma + 2Q\sin\gamma - \frac{qR}{4}\sin\gamma\cos\gamma\right\} + \theta_1 (A_1 - B_1) - \theta (A - B).$$

Again, (IX a) and (IX b) must be identical for $\varphi = \beta$; therefore

$$A_1 - B_1 = -\frac{qR}{4}(\beta + 2\beta\sin^2\beta + 3\sin\beta\cos\beta) = -\frac{qR}{4}\mu,$$

if

$$\mu = \beta + 2\beta\sin^2\beta + 3\sin\beta\cos\beta.$$

The arch being fixed at the piers, we must have $\varphi_1 - \varphi = 0$ for $\varphi = \alpha$ in (9 a), and $\varphi_1 - \varphi = 0$ for $\varphi = -\alpha$ in (9 b); hence

$$A - B = -2\alpha\left(N\sin\alpha - P\sin\alpha - Q\cos\alpha + \frac{qR}{2}\sin^2\alpha + \frac{qR}{8} - \frac{qR}{4}\sin^2\beta\right) - qR\sin\beta\cos\alpha - 2Q\sin\alpha + \frac{qR}{4}\sin\alpha\cos\alpha.$$

By introducing these values of $(A_1 - B_1)$ and $(A - B)$ into equation (10), and for brevity putting

$$\sigma = \frac{\theta}{\theta_1}\alpha + \frac{\theta_1 - \theta}{\theta_1}\gamma, \qquad \sigma_2 = \frac{\theta}{\theta_1}\sin\alpha + \frac{\theta_1 - \theta}{\theta_1}\sin\gamma,$$

$$\sigma_1 = \frac{\theta}{\theta_1}\sin\alpha\cos\alpha + \frac{\theta_1 - \theta}{\theta_1}\sin\gamma\cos\gamma, \qquad \sigma_3 = \frac{\theta}{\theta_1}\cos\alpha + \frac{\theta_1 - \theta}{\theta_1}\cos\gamma,$$

we deduce

$$(11)\quad N\sin\alpha = P\sin\alpha + Q\left(\cos\alpha - \frac{\sigma_2}{\sigma}\right) + qR\left(\frac{\sin^2\beta}{4} - \frac{\sigma_3}{\sigma}\cdot\frac{\sin\beta}{2} + \frac{\mu}{8\sigma}\right) + qR\left(-\frac{1}{8} - \frac{\sin^2\alpha}{2} + \frac{\sigma_1}{8\sigma}\right).$$

Substituting this value of $N\sin\alpha$ in the equations (9) and (IX), and for brevity putting

$$G = -Q\cdot\frac{\sigma_2}{\sigma} + qR\left(-\frac{\sigma_3}{\sigma}\cdot\frac{\sin\beta}{2} + \frac{\mu}{8\sigma} + \frac{\sigma_1}{8\sigma}\right)$$

$$H = qR\left(\frac{1}{8} + \frac{\sin^2\beta}{4}\right),$$

these equations become

(12 a)

$$\varphi_1 - \varphi = \frac{R^2}{\epsilon\theta_1}\left\{(G+H)\varphi - (P-q\alpha)\cos\varphi + Q\sin\varphi - \frac{qR}{4}\sin\varphi\cos\varphi + A\right\},$$

(XII a)

$$\varphi_1 - \varphi = \frac{R^2}{\epsilon\theta}\left\{(G+H)\varphi - (P-q\alpha)\cos\varphi + Q\sin\varphi - \frac{qR}{4}\sin\varphi\cos\varphi + A_1\right\},$$

(XII b)

$$\varphi_1 - \varphi = \frac{R^2}{\epsilon\theta}\left\{(G-H)\varphi - (P-q\alpha + qR\sin\beta)\cos\varphi + Q\sin\varphi + B_1\right\},$$

(12 b)

$$\varphi_1 - \varphi = \frac{R^2}{\epsilon\theta_1}\left\{(G-H)\varphi - (P-q\alpha + qR\sin\beta)\cos\varphi + Q\sin\varphi + B\right\}.$$

Now, since we have for the undeflected arc s and the deflected arc s_1,

$$dx = ds\cos\varphi, \qquad dx_1 = ds_1\cos\varphi_1,$$
$$dy = ds\sin\varphi, \qquad dy_1 = ds_1\sin\varphi_1,$$

and $ds = ds_1 = -Rd\varphi$, there follows very nearly

$$dx_1 - dx = -Rd\varphi\,(\cos\varphi_1 - \cos\varphi) = Rd\varphi\,(\varphi_1 - \varphi)\sin\varphi,$$
$$dy_1 - dy = -Rd\varphi\,(\sin\varphi_1 - \sin\varphi) = -Rd\varphi\,(\varphi_1 - \varphi)\cos\varphi.$$

Hence multiplying the equations (12) and (XII) by $Rd\varphi\sin\varphi$ and integrating,* we find for the different parts of the arch the horizontal deflections:

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* Employing the known integrals already used, and also

$\int d\phi\,.\,\phi\sin\phi = \sin\phi - \phi\cos\phi$, $\int d\phi\,.\,\phi\cos\phi = \cos\phi + \phi\sin\phi$,
$\int d\phi\,.\,\sin\phi\cos\phi = \frac{1}{2}\sin^2\phi$, $\int d\phi\,,\cos^2\phi = \frac{1}{2}(\phi + \sin\phi\cos\phi)$,
$\int d\phi\sin^2\phi\cos\phi = \frac{1}{3}\sin^3\phi$, $\int d\phi\,,\cos^2\phi\sin\phi = -\frac{1}{3}\cos^3\phi$.

$$(13\,a)\quad x_1-x=\frac{R^3}{\varepsilon\theta_1}\Big\{(G+H)(\sin\varphi-\varphi\cos\varphi)-(P-qa)\frac{\sin^2\varphi}{2}$$
$$+Q.\frac{\varphi-\sin\varphi\cos\varphi}{2}-\frac{qR}{12}\sin^3\varphi-A\cos\varphi+C\Big\},$$

$$(\text{XIII a})\quad x_1-x=\frac{R^3}{\varepsilon\theta}\Big\{(G+H)(\sin\varphi-\varphi\cos\varphi)-(P-qa)\frac{\sin^2\varphi}{2}$$
$$+Q.\frac{\varphi-\sin\varphi\cos\varphi}{2}-\frac{qR}{12}\sin^3\varphi-A_1\cos\varphi+C_1\Big\},$$

$$(\text{XIII b})\quad x_1-x=\frac{R^3}{\varepsilon\theta}\Big\{(G-H)(\sin\varphi-\varphi\cos\varphi)-(P-qa+qR\sin\beta)\frac{\sin^2\varphi}{2}$$
$$+Q.\frac{\varphi-\sin\varphi\cos\varphi}{2}-B_1\cos\varphi+D_1\Big\},$$

$$(13\,b)\quad x_1-x=\frac{R^3}{\varepsilon\theta_1}\Big\{(G-H)(\sin\varphi-\varphi\cos\varphi)-(P-qa+qR\sin\beta)\frac{\sin^2\varphi}{2}$$
$$+Q.\frac{\varphi-\sin\varphi\cos\varphi}{2}-B\cos\varphi+D\Big\}.$$

And multiplying (12) and (XII) by $-Rd\varphi\cos\varphi$ and integrating, we find the corresponding vertical deflections:

$$(14\,a)\quad y_1-y=\frac{R^3}{\varepsilon\theta_1}\Big\{-(G+H)(\cos\varphi+\varphi\sin\varphi)$$
$$+(P-qa).\frac{\varphi+\sin\varphi\cos\varphi}{2}$$
$$-Q.\frac{\sin^2\varphi}{2}-\frac{qR}{12}\cos^3\varphi-A\sin\varphi+E\Big\},$$

$$(\text{XIV a})\quad y_1-y=\frac{R^3}{\varepsilon\theta}\Big\{-(G+H)(\cos\varphi+\varphi\sin\varphi)$$
$$+(P-qa).\frac{\varphi+\sin\varphi\cos\varphi}{2}$$
$$-Q.\frac{\sin^2\varphi}{2}-\frac{qR}{12}\cos^3\varphi-A_1\sin\varphi+E_1\Big\},$$

$$(\text{XIV b})\quad y_1-y=\frac{R^3}{\varepsilon\theta}\Big\{-(G-H)(\cos\varphi+\varphi\sin\varphi)$$
$$+(P-qa+qR\sin\beta)\frac{\varphi+\sin\varphi\cos\varphi}{2}$$
$$-Q.\frac{\sin^2\varphi}{2}-B_1\sin\varphi+F_1\Big\},$$

$$(14\,b)\quad y_1-y=\frac{R^3}{\varepsilon\theta_1}\Big\{-(G-H)(\cos\varphi+\varphi\sin\varphi)$$
$$+(P-qa+qR\sin\beta)\frac{\varphi+\sin\varphi\cos\varphi}{2}$$
$$-Q.\frac{\sin^2\varphi}{2}-B\sin\varphi+F\Big\}.$$

Since (13 a) and (XIII a) are identical for $\varphi=\gamma$, and (13 b) and (XIII b) are identical for $\varphi=-\gamma$, we deduce

$$0=(\theta_1-\theta)\Big\{(G+H)(\sin\gamma-\gamma\cos\gamma)-(P-qa)\frac{\sin^2\gamma}{2}$$
$$+Q.\frac{\gamma-\sin\gamma\cos\gamma}{2}-\frac{qR}{12}\sin^2\gamma\Big\}$$
$$-(\theta_1A_1-\theta A)\cos\gamma+\theta_1C_1-\theta C,$$
$$0=(\theta_1-\theta)\Big\{-(G-H)(\sin\gamma-\gamma\cos\gamma)-(P-qa+qR\sin\beta)\frac{\sin^2\gamma}{2}$$
$$-Q.\frac{\gamma-\sin\gamma\cos\gamma}{2}\Big\}-(\theta_1B_1-\theta B)\cos\gamma+\theta_1D_1-\theta D;$$

and from these, by subtraction,

$$(15)\quad 0=(\theta_1-\theta)\Big\{2G(\sin\gamma-\gamma\cos\gamma)+\frac{qR\sin\beta\sin^2\gamma}{2}$$
$$+Q.(\gamma-\sin\gamma\cos\gamma)-\frac{qR}{12}\sin^2\gamma\Big\}$$
$$-\theta_1(A_1-B_1)\cos\gamma+\theta(A-B)\cos\gamma+\theta_1(C_1-D_1)-\theta(C-D).$$

The equations (XIII a) and (XIII b) are identical for $\varphi=\beta$; and hence

$$0=2H(\sin\beta-\beta\cos\beta)+\frac{qR}{2}\sin^2\beta-\frac{qR}{12}\sin^2\beta-(A_1-B_1)\cos\beta+C_1-D_1.$$

Introducing the values of H and (A_1-B_1) before given (p. 13 & 12), we deduce

$$C_1-D_1=-qR\Big(\sin\beta+\frac{\sin^3\beta}{6}\Big)=-qR\nu,$$

if

$$\nu=\sin\beta+\frac{\sin^3\beta}{6}.$$

The equation (13 a) gives $x_1-x=0$ for $\varphi=\alpha$; and (13 b) gives $x_1-x=0$ for $\varphi=-\alpha$; and hence the difference of these equations, when these substitutions are made, gives

$$C-D=-2G(\sin\alpha-\alpha\cos\alpha)-\frac{qR\sin\beta\sin^2\alpha}{2}-Q(\alpha-\sin\alpha\cos\alpha)$$
$$+\frac{qR}{12}\sin^2\alpha+(A-B)\cos\alpha.$$

The equation (12 a) gives $\varphi_1-\varphi=0$ for $\varphi=\alpha$; and (12 b) gives $\varphi_1-\varphi=0$ for $\varphi=-\alpha$; and hence the difference of the equations, these substitutions being made, gives

$$A-B=-2G\alpha-qR\sin\beta\cos\alpha-2Q\sin\alpha+\frac{qR}{4}\sin\alpha\cos\alpha,$$

which, substituted in the preceding value of $C-D$, gives

$$C-D=-2G\sin\alpha-qR\sin\beta\left(1-\frac{\sin^2\alpha}{2}\right)-Q(\alpha+\sin\alpha\cos\alpha)+qR\left(\frac{\sin\alpha}{4}-\frac{\sin^3\alpha}{6}\right)$$

The equation (15) becomes by substituting the values of (A_1-B_1), (C_1-D_1), $(A-B)$ and $(C-D)$,

$$\begin{aligned}0=&2G\left\{(\theta_1-\theta)(\sin\gamma-\gamma\cos\gamma)+\theta(\sin\alpha-\alpha\cos\gamma)\right\}\\&+Q\left\{(\theta_1-\theta)(\gamma-\sin\gamma\cos\gamma)+\theta(\alpha+\sin\alpha\cos\alpha-2\sin\alpha\cos\gamma)\right\}\\&+qR\sin\beta\left\{(\theta_1-\theta)\frac{\sin^2\gamma}{2}+\theta\left(1-\cos\alpha\cos\gamma-\frac{\sin^2\alpha}{2}\right)\right\}\\&+qR\left\{-(\theta_1-\theta)\frac{\sin^2\gamma}{12}+\theta\left(\frac{\sin\alpha\cos\alpha\cos\gamma}{4}-\frac{\sin\alpha}{4}+\frac{\sin^3\alpha}{6}\right)\right\}\\&+qR\theta_1\left(\frac{\cos\gamma}{4}\mu-\nu\right).\end{aligned}$$

If in this equation we substitute for G and ν their values, and then for brevity put

$$\sigma_4=\frac{\theta}{\theta_1}\cdot\frac{\sin^2\alpha}{2}+\frac{\theta_1-\theta}{\theta_1}\cdot\frac{\sin^2\gamma}{2},$$
$$\sigma_5=\frac{\theta}{\theta_1}\cdot\frac{\sin^2\alpha}{6}+\frac{\theta_1-\theta}{\theta_1}\cdot\frac{\sin^3\gamma}{6};$$

and with these, together with the quantities σ, σ_1, σ_2, σ_3 before introduced, compute the following quantities,

$$\begin{aligned}\varkappa&=\sigma+\sigma_1-\frac{2\sigma_2^2}{\sigma}\\\varkappa_1&=\frac{\sigma_2\sigma_3}{\sigma}+\sigma_4\\\varkappa_2&=\frac{\sigma_2}{4\sigma},\\\varkappa_3&=\varkappa_2(\sigma-\sigma_1)-\sigma_5,\end{aligned}$$

we shall obtain the following formula for Q.

(16) $$Q=\frac{qR}{\varkappa}(\varkappa_1\sin\beta-\varkappa_2\mu+\varkappa_3+\tfrac{1}{6}\sin^3\beta).$$

The equations (14 a) and (XIV a) are identical for $\varphi=\gamma$, while (14 b) and (XIV b) are identical for $\varphi=-\gamma$; hence, making these substitutions, we deduce

$$0=(\theta_1-\theta)\left\{(G+H)(\cos\gamma+\gamma\sin\gamma)-(P-q\alpha)\frac{\gamma+\sin\gamma\cos\gamma}{2}+Q\frac{\sin^2\gamma}{2}\right.$$
$$\left.+\frac{qR}{12}\cos^3\gamma\right\}+(\theta_1 A_1-\theta A)\sin\gamma-(\theta_1 E_1-\theta E),$$
$$0=(\theta_1-\theta)\left\{(G-H)(\cos\gamma+\gamma\sin\gamma)+(P-q\alpha+qR\sin\beta)\frac{\gamma+\sin\gamma\cos\gamma}{2}\right.$$
$$\left.+Q\frac{\sin^2\gamma}{2}\right\}-(\theta_1 B_1-\theta B)\sin\gamma-(\theta_1 F_1-\theta F);$$

and from these, by subtraction,

(17)
$$0=(\theta_1-\theta)\left\{2H(\cos\gamma+\gamma\sin\gamma)-\left(P-q\alpha+\frac{qR\sin\beta}{2}\right)(\gamma+\sin\gamma\cos\gamma)\right.$$
$$\left.+\frac{qR}{12}\cos^3\gamma\right\}+\theta_1(A_1+B_1)\sin\gamma-\theta(A+B)\sin\gamma$$
$$-\theta_1(E_1-F_1)+\theta(E-F).$$

The equations (XIV a) and (XIV b) are identical for $\varphi=\beta$; hence

$$0=2H(\cos\beta+\beta\sin\beta)+\frac{qR\sin\beta}{2}(\beta+\sin\beta\cos\beta)+\frac{qR}{12}\cos^3\beta$$
$$+(A_1-B_1)\sin\beta-(E_1-F_1),$$

which, when the previous values of H and (A_1-B_1) are substituted, gives

$$E_1-F_1=qR\left(\frac{\cos\beta}{2}+\frac{\beta\sin\beta}{2}-\frac{\cos^3\beta}{6}\right)=qR\,\xi,$$

if

$$\xi=\frac{\cos\beta}{2}+\frac{\beta\sin\beta}{2}-\frac{\cos^3\beta}{6}.$$

If we take the equations (p. 12) from which (10) was deduced by subtraction, and now add them, we find

$$0=(\theta_1-\theta)\left\{2H\gamma-2\left(P-q\alpha+\frac{qR\sin\beta}{2}\right)\cos\gamma-\frac{qR}{4}\sin\gamma\cos\gamma\right\}$$
$$+\theta_1(A_1+B_1)-\theta(A+B).$$

Multiplying this by $\sin\gamma$, and subtracting it from (17), we have

$$(18)\quad 0=(\theta_1-\theta)\left\{2H\cos\gamma-(P-q\alpha+qR\sin\beta)(\gamma-\sin\gamma\cos\gamma)\right.$$
$$\left.+qR\left(\frac{\cos\gamma}{4}-\frac{\cos^3\gamma}{6}\right)\right\}-\theta_1(E_1-F_1)+\theta(E-F).$$

Since (14 a) must give $y_1 - y = 0$ for $\varphi = \alpha$, and (14 b) must give $y_1 - y = 0$ for $\varphi = -\alpha$, their difference, when these substitutions are made, is

$$0 = 2H(\cos\alpha + \alpha\sin\alpha) - \left(P - q\alpha + \frac{qR\sin\beta}{2}\right)(\alpha + \sin\alpha\cos\alpha) + \frac{qR}{12}\cos^3\alpha + (A+B)\sin\alpha - (E-F)$$

and the same substitutions being made in (12 a) and (12 b), their sum gives

$$0 = 2H\alpha - 2\left(P - q\alpha + \frac{qR\sin\beta}{2}\right)\cos\alpha - \frac{qR}{4}\sin\alpha\cos\alpha + A + B.$$

and these last two equations combined give

$$E - F = 2H\cos\alpha - \left(P - q\alpha + \frac{qR\sin\beta}{2}\right)(\alpha - \sin\alpha\cos\alpha) + qR\left(\frac{\cos\alpha}{4} - \frac{\cos^3\alpha}{6}\right)$$

With this value of $(E-F)$ and the values of $(E_1 - F_1)$ and H, the equation (18) now becomes

$$0 = (\theta_1 - \theta)\left\{\frac{qR}{2}\sin^2\beta\cos\gamma + qR\left(\frac{\cos\gamma}{2} - \frac{\cos^3\gamma}{6}\right)\right\} + \theta\left\{\frac{qR}{2}\sin^2\beta\cos\alpha + qR\left(\frac{\cos\alpha}{2} - \frac{\cos^3\alpha}{6}\right)\right\} - \theta_1 qR\xi - \left(P - q\alpha + \frac{qR\sin\beta}{2}\right)\left\{\theta(\alpha - \sin\alpha\cos\alpha) + (\theta_1 - \theta)(\gamma - \sin\gamma\cos\gamma)\right\},$$

and from this, by introducing the following additional auxiliary

$$\sigma_6 = \frac{\theta}{\theta_1}\cdot\frac{\cos^3\alpha}{3} + \frac{\theta_1 - \theta}{\theta_1}\cdot\frac{\cos^3\gamma}{3},$$

we find for the vertical resistance of the left pier the formula

$$(19)\quad P = qR\left\{\frac{\sigma^3}{\sigma - \sigma_1}\cdot\frac{\sin^2\beta}{2} - \frac{\sin\beta}{2} - \frac{\xi}{\sigma - \sigma_1} + \frac{\sigma_3 - \sigma_6}{2(\sigma - \sigma_1)} + \sin\alpha\right\}.$$

The values of Q and P being computed by (16) and (19), we can then find N (and $N\alpha = NR\sin\alpha$) by means of (11), and finally the resistances P_1, Q_1, $N_1\alpha$ at the right pier by means of the equations (6).

For the calculations of the deflections by the equations (13), (XIII), (14) and (XIV), we have yet to determine the constants A, B, C, etc.

We have already found (p. 18 and p. 15),

$$A+B=-2H\alpha+2\left(P-q\alpha+\frac{qR\sin\beta}{2}\right)\cos\alpha+\frac{qR}{4}\sin\alpha\cos\alpha,$$

$$A-B=-2G\alpha-qR\sin\beta\cos\alpha-2Q\sin\alpha+\frac{qR}{4}\sin\alpha\cos\alpha,$$

from which A and B are immediately deduced.

By putting $\varphi=\alpha$ in (13 a), and $\varphi=-\alpha$ in (13 b), and adding the equations (the first members being then zero), we obtain an equation involving $(A+B)$ and $(C+D)$, and by substituting the preceding value of $(A+B)$, the value of $(C+D)$ is obtained; and thus with the value of $(C-D)$ already found (p. 16), we have

$$C+D=-2H\sin\alpha+2\left(P-q\alpha+\frac{qR\sin\beta}{2}\right)\left(1-\frac{\sin^2\alpha}{2}\right)+qR\left(\frac{\sin\alpha}{4}-\frac{\sin^3\alpha}{6}\right),$$

$$C-D=-2G\sin\alpha-qR\sin\beta\left(1-\frac{\sin^2\alpha}{2}\right)-Q(\alpha+\sin\alpha\cos\alpha)+qR\left(\frac{\sin\alpha}{4}-\frac{\sin^3\alpha}{6}\right),$$

from which C and D are found.

By putting $\varphi=\alpha$ in (14 a) and $\varphi=-\alpha$ in (14 b) and adding the equations (the first members being then zero), we obtain a relation between $(A-B)$ and $(E+F)$, in which we have only to substitute the preceding value of $(A-B)$ to obtain $(E+F)$. This with the value of $(E-F)$ from p. 18, gives

$$E+F=2G\cos\alpha+\frac{qR\sin\beta}{2}(\alpha-\sin\alpha\cos\alpha)-Q\sin^2\alpha+qR\left(\frac{\cos\alpha}{4}-\frac{\cos^3\alpha}{6}\right),$$

$$E-F=2H\cos\alpha-\left(P-q\alpha+\frac{qR\sin\beta}{2}\right)(\alpha-\sin\alpha\cos\alpha)+qR\left(\frac{\cos\alpha}{4}-\frac{\cos^3\alpha}{6}\right),$$

whence E and F are found.

We have, further, from equations on p. 17 and p. 12,

$$A_1+B_1=\frac{\theta}{\theta_1}(A+B)+\frac{\theta_1-\theta}{\theta_1}\left\{-2H\gamma+2\left(P-q\alpha+\frac{qR\sin\beta}{2}\right)\cos\gamma+\frac{qR}{4}\sin\gamma\cos\gamma\right\},$$

$$A_1-B_1=-\frac{qR}{4}\mu=-\frac{qR}{4}(\beta+2\beta\sin^2\beta+3\sin\beta\cos\beta),$$

by which A_1 and B_1 are immediately found.

If we add the two equations on p. 15, from which (15) was deduced, and in the result substitute the above value of (A_1+B_1), we shall find the following expression for (C_1+D_1); and the value of (C_1-D_1) has already been found on p. 15; thus we have

$$C_1+D_1=\frac{\theta}{\theta_1}(C+D)+\frac{\theta_1-\theta}{\theta_1}\Big\{-2H\sin\gamma$$
$$+2\Big(P-qa+\frac{qR\sin\beta}{2}\Big)\Big(1-\frac{\sin^2\gamma}{2}\Big)+qR\Big(\frac{\sin\gamma}{4}-\frac{\sin^3\gamma}{6}\Big)\Big\},$$
$$C_1-D_1=-qR\nu=-qR\,(\sin\beta+\tfrac{1}{6}\sin^3\beta),$$

from which C_1 and D_1 are found.

By adding the two equations on p. 17 from which (17) was deduced, we obtain the following expression for (E_1+F_1); and (E_1-F_1) has already been found on p. 17; thus we have

$$E_1+F_1=(A_1-B_1)\sin\gamma-\frac{\theta}{\theta_1}(A-B)\sin\gamma+\frac{\theta}{\theta_1}(E+F)$$
$$+\frac{\theta_1-\theta}{\theta_1}\Big\{2G(\cos\gamma+\gamma\sin\gamma)+\frac{qR\sin\beta}{2}(\gamma+\sin\gamma\cos\gamma)$$
$$+Q\sin^2\gamma+\frac{qR}{12}\cos^3\gamma\Big\},$$
$$E_1-F_1=qR\xi=qR\Big(\frac{\cos\beta}{2}+\frac{\beta\sin\beta}{2}-\frac{\cos^3\beta}{6}\Big),$$

from which E_1 and F_1 are found.

The expressions for (A_1+B_1), (C_1+D_1), (E_1+F_1), may also be put under the following forms:

$$A_1+B_1=-2H\sigma+2\Big(P-qa+\frac{qR\sin\beta}{2}\Big)\sigma_3+\frac{qR}{4}\cdot\sigma_1,$$
$$C_1+D_1=-2H\sigma_2+2\Big(P-qa+\frac{qR\sin\beta}{2}\Big)(1-\sigma_4)+qR\Big(\frac{\sigma_2}{4}-\sigma_5\Big),$$
$$E_1+F_1=-2Q\varkappa_1+\frac{qR}{4}\cdot\frac{\sigma_3}{\sigma}\cdot\mu+qR\sin\beta\Big(\frac{\sigma-\sigma_1}{2}-\frac{\sigma_3{}^2}{\sigma}\Big)$$
$$+\frac{qR}{4}\Big(\frac{\sigma_1\sigma_3}{\sigma}+\sigma_3-2\sigma_6\Big),$$

and a good verification of the correctness of a large part of the whole work is obtained by computing these quantities under both forms.

Special Investigation for the Case of a Load equally distributed over the whole Arch.

In the preceding investigation we supposed $\gamma>\beta>-\gamma$; that is, end piece entirely loaded, and the other entirely unloaded. We

cannot then apply our formulæ to the case $\beta = -\alpha$, which represents a load covering the whole arch. A special investigation for this case is therefore necessary.

Since in this case all is symmetrical with respect to the centre of the arch, we need to consider only one half of the arch, consisting of one end piece (from $\varphi = \alpha$ to $\varphi = \gamma$) and one centre piece (from $\varphi = \gamma$ to $\varphi = 0$). They are both loaded and we can use for them the equations (9 a) and (IX a). The whole load will be equally divided between the two piers, and therefore

$$P = P_1 = q\alpha = qR \sin \alpha .$$

Hence our equations will be

$$(20) \qquad \varphi_1 - \varphi = \frac{R^2}{\varepsilon\theta_1} \left\{ \left(N \sin\alpha - Q\cos\alpha - \frac{qR}{2}\sin^2\alpha + \frac{qR}{4} \right)\varphi + Q\sin\varphi - \frac{qR}{4}\sin\varphi\cos\varphi + A \right\},$$

$$(XX) \qquad \varphi_1 - \varphi = \frac{R^2}{\varepsilon\theta} \left\{ \left(N \sin\alpha - Q\cos\alpha - \frac{qR}{2}\sin^2\alpha + \frac{qR}{4} \right)\varphi + Q\sin\varphi - \frac{qR}{4}\sin\varphi\cos\varphi + A_1 \right\}.$$

Since in (20) we have $\varphi_1 - \varphi = 0$ for $\varphi = \alpha$, we find

$$A = -\left(N\sin\alpha - Q\cos\alpha - \frac{qR}{2}\sin^2\alpha + \frac{qR}{4} \right)\alpha - Q\sin\alpha + \frac{qR}{4}\sin\alpha\cos\alpha,$$

and since in (XX) $\varphi_1 - \varphi = 0$ for $\varphi = 0$, we have

$$A_1 = 0.$$

And since the two equations must be identical for $\varphi = \gamma$, we deduce

$$0 = [\theta\alpha + (\theta_1 - \theta)\gamma] \left\{ N\sin\alpha - Q\cos\alpha - \frac{qR}{2}\sin^2\alpha + \frac{qR}{4} \right\}$$
$$+ [\theta\sin\alpha + (\theta_1 - \theta)\sin\gamma]\, Q - [\theta\sin\alpha\cos\alpha + (\theta_1 - \theta)\sin\gamma\cos\gamma]\frac{qR}{4}$$

whence, with the notation before used (p. 12),

$$(21) \qquad N\sin\alpha = Q\left(\cos\alpha - \frac{\sigma_2}{\sigma}\right) + qR\left(-\frac{1}{4} + \frac{\sin^2\alpha}{2} + \frac{\sigma_1}{4\sigma}\right).$$

Substituting this value, our equations become

$$(22) \quad \varphi_1 - \varphi = \frac{R^2}{\varepsilon\theta_1}\left\{ \left(\frac{qR}{4}\cdot\frac{\sigma_1}{\sigma} - Q\cdot\frac{\sigma_2}{\sigma} \right)\varphi + Q\sin\varphi - \frac{qR}{4}\sin\varphi\cos\varphi + A \right\},$$

$$(XXII) \quad \varphi_1 - \varphi = \frac{R^2}{\varepsilon\theta}\left\{ \left(\frac{qR}{4}\cdot\frac{\sigma_1}{\sigma} - Q\cdot\frac{\sigma_2}{\sigma} \right)\varphi + Q\sin\varphi - \frac{qR}{4}\sin\varphi\cos\varphi \right\}$$

FOR LOADED PART.

$$M = Na - Pa \,.\, m + \frac{qa^2}{2} m^2 + Qy.$$

FOR UNLOADED PART.

$$M = Na - Pa \,.\, m + qna^2 \left(m - \frac{n}{2} \right) + Qy.$$

The load will be taken for the cases in which it covers $\frac{1}{8}$, $\frac{2}{8}$, $\frac{3}{8}$, &c., of the span, up to $\frac{8}{8}$ or the whole span; the corresponding values of n being $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, &c., up to $\frac{8}{4}$ or 2.

The strains on each member resulting from the moment M will follow from the equation

$$T = \mp \frac{M}{\delta},$$

δ being the distance of the centres of the members. The $\left\{ \begin{matrix} \text{upper} \\ \text{lower} \end{matrix} \right\}$ sign will be used for the $\left\{ \begin{matrix} \text{upper} \\ \text{lower} \end{matrix} \right\}$ member; and positive values of T will express compressive strain, negative values tensile strain.

The values of T for the permanent load will be found from the values previously found for the movable load with $n = 2$, by multiplying these values by the ratio $\frac{q'}{q}$; q being the movable load, and q' the permanent load per linear foot of the span.

Strains in the direction of the Arch, and Shearing Strains.

On p. 5 we separated the total force into a moment of flexure M, and a force S acting at the centre of the arch. The force S was further resolved into its components S' and S'', the former lying in the direction of a tangent to the central curve of the arch, the latter in the direction of the normal. We distinguish S' as the *strain in the direction of the arch*, S'' as the *shearing strain.*

If V is the vertical force acting at any point (x, y), then, φ being the angle which the tangent to the curve at this point makes with the horizontal line, we have

$$S' = Q \cos \varphi + V \sin \varphi,$$
$$S'' = Q \sin \varphi - V \cos \varphi,$$

and the value of V will be

For loaded part, $V = P - qx = P - qma$;
For unloaded part, $V = P - qna$.

The value of S', from the nature of the case, will always be positive, and be compressive strain.

The value of S'' will be positive in some parts of the arch and negative in others. The positive value will be a strain towards the centre; the negative value a strain from the centre. As in our formulæ we reckon from the pier to which P belongs, S'' is the shearing strain of that segment of the arch which extends from that pier to the point at which the strain is considered to exist.

Since the movable loads may extend from either end of the bridge, all the strains computed for one half the arch must also be applied to the other half in deducing the necessary sectional areas of the members and braces, as will be illustrated hereafter.

The values of S' and S'' for permanent load will be found by the same formulæ, taking the proper values of Q and P for this case (p. 23), and $V = P - q'ma$, q' being the permanent load per linear foot of the span.

Effect of Changes of Temperature upon the Arch.

If the rib was allowed to expand and contract freely, no strains would be produced by a change of temperature. The figures of the arch assumed at different temperatures would be similar; and if we denote by λ the linear expansion for a given change of temperature, of a bar of the same material as that composing the rib, and of the length $= 1$, the linear dimensions of any portion of the rib, after the change of temperature has taken place, would be expressed by multiplying its dimensions before the change by $1+\lambda$. The span of the rib before the change being denoted by $2a$, it would after the change, no obstacle existing to its expansion, be $2a(1+\lambda)$. But the piers prevent this change by the resistances which they exert; and the resulting curve of the rib is that which would result if to the expanded arch of the span $2a(1+\lambda)$, resistances were applied which reduce this span to $2a$. But since the change $2a\lambda$ is very small compared with the whole span, it will be sufficiently accurate to investigate the amount of the resistances which would reduce the original arch of the span $2a$ to one of the span $2a(1-\lambda)$. This somewhat simplifies the investigation, which is necessarily but approximative in its nature; especially as we regard the modulus of elasticity of steel (ε) as constant under the different temperatures considered, whereas it is doubtless subject to changes which would affect our results by minute fractions of the same order as those which we neglect in the method here adopted.

Since the weight of the bridge is not changed by the temperature, the resistances at the pier may be considered as consisting of a horizontal force Q and a moment Na. The moment of flexure for any point (x, y) of the deflected arch is, then,

$$M = Na + Qy = NR\sin\alpha + QR(\cos\varphi - \cos\alpha);$$

and since our equations (VII) and (7) are general, we shall have

FOR END PIECE,

$$(27) \qquad d\varphi_1 - d\varphi = \frac{R^2}{\epsilon\theta_1}(N\sin\alpha - Q\cos\alpha + Q\cos\varphi)\,d\varphi;$$

FOR CENTRE PIECE,

$$(\text{XXVII}) \qquad d\varphi_1 - d\varphi = \frac{R^2}{\epsilon\theta}(N\sin\alpha - Q\cos\alpha + Q\cos\varphi)\,d\varphi.$$

Integrating:

$$(28) \qquad \varphi_1 - \varphi = \frac{R^2}{\epsilon\theta_1}\left\{(N\sin\alpha - Q\cos\alpha)\varphi + Q\sin\varphi + A\right\},$$

$$(\text{XXVIII}) \qquad (\varphi_1 - \varphi) = \frac{R^2}{\epsilon\theta}\left\{(N\sin\alpha - Q\cos\alpha)\varphi + Q\sin\varphi\right\};$$

no constant being required in the last equation, since at the centre of the arch we must have $\varphi_1 - \varphi = 0$ for $\varphi = 0$. Since from (28) we must have $\varphi_1 - \varphi = 0$ for $\varphi = \alpha$, we deduce

$$A = -(N\sin\alpha - Q\cos\alpha)\,\alpha - Q\sin\alpha;$$

and since (28) and (XXVIII) are identical for $\varphi = \gamma$, we deduce

$$0 = (\theta_1 - \theta)\{(N\sin\alpha - Q\cos\alpha)\gamma + Q\sin\gamma\} - \theta A.$$

Substituting in this the preceding value of A, and introducing the symbols σ and σ_2 with the significations adopted on p. 12, we find

$$(29) \qquad N\sin\alpha = -\left(\frac{\sigma_2}{\sigma} - \cos\alpha\right)Q,$$

by means of which our equations now become

$$(30) \qquad \varphi_1 - \varphi = \frac{R^2}{\epsilon\theta_1}\cdot Q\left(\sin\varphi - \frac{\sigma_2}{\sigma}\varphi + \frac{\sigma_2}{\sigma}\alpha - \sin\alpha\right),$$

$$(\text{XXX}) \qquad \varphi_1 - \varphi = \frac{R^2}{\epsilon\theta}\cdot Q\left(\sin\varphi - \frac{\sigma_2}{\sigma}\varphi\right).$$

Multiplying these by $Rd\varphi\sin\varphi$ and integrating, we deduce, as on pp. 13 and 14.

$$(31) \qquad x_1 - x = \frac{R^3}{\epsilon\theta_1}\cdot Q\cdot\left\{\frac{\varphi - \sin\varphi\cos\varphi}{2} - \frac{\sigma_2}{\sigma}(\sin\varphi - \varphi\cos\varphi) - \left(\frac{\sigma_2\alpha}{\sigma} - \sin\alpha\right)\cos\varphi + C\right\},$$

$$(\text{XXXI}) \qquad x_1 - x = \frac{R^3}{\epsilon\theta}Q\cdot\left\{\frac{\varphi - \sin\varphi\cos\varphi}{2} - \frac{\sigma_2}{\sigma}(\sin\varphi - \varphi\cos\varphi)\right\},$$

no constant being necessary in the last equation, since we must have $x_1 - x = 0$, for $\varphi = 0$.

Also multiplying (30) and (xxx) by $-Rd\varphi\cos\varphi$ and integrating, we obtain

$$(32)\qquad y_1-y=\frac{R^3}{\varepsilon\theta_1}\cdot Q\left\{-\frac{\sin^2\varphi}{2}+\frac{\sigma_2}{\sigma}(\cos\varphi+\varphi\sin\varphi)-\left(\frac{\sigma_2\alpha}{\sigma}-\sin\alpha\right)\sin\varphi+E\right\},$$

$$(\text{xxxii})\quad y_1-y=\frac{R^3}{\varepsilon\theta}\cdot Q\left\{-\frac{\sin^2\varphi}{2}+\frac{\sigma_2}{\sigma}(\cos\varphi+\varphi\sin\varphi)+E_1\right\}.$$

Since the resistances of the piers are supposed to change the length of the span by the quantity $2\lambda a$, each end of the arch is displaced horizontally by the quantity λa; that is, the equation (31) must give $x_1-x=\lambda a$ when $\varphi=\alpha$; hence

$$\lambda a=\frac{R^3}{\varepsilon\theta_1}\cdot Q\cdot\left\{\frac{\alpha+\sin\alpha\cos\alpha}{2}-\frac{\sigma_2}{\sigma}\sin\alpha+C\right\}.$$

The equations (31) and (xxxi) being identical for $\varphi=\gamma$, we deduce

$$C=\frac{\theta_1-\theta}{\theta}\left\{\frac{\gamma-\sin\gamma\cos\gamma}{2}-\frac{\sigma_2}{\sigma}(\sin\gamma-\gamma\cos\gamma)\right\}+\left(\frac{\sigma_2}{\sigma}\alpha-\sin\alpha\right)\cos\gamma,$$

which can be reduced to the form

$$C=\frac{\theta_1-\theta}{\theta}\left\{\frac{\gamma+\sin\gamma\cos\gamma}{2}-\frac{\sigma_2}{\sigma}\sin\gamma\right\},$$

and this substituted in the equation for λa, gives (by bringing θ outside the brackets and putting θ_1 within them),

$$\lambda a=\frac{R^3}{\varepsilon\theta}\cdot Q\left\{\frac{\theta}{\theta_1}\cdot\frac{\alpha+\sin\alpha\cos\alpha}{2}+\frac{\theta_1-\theta}{\theta_1}\cdot\frac{\gamma+\sin\gamma\cos\gamma}{2}-\frac{\sigma_2}{\sigma}\left(\frac{\theta}{\theta_1}\sin\alpha+\frac{\theta_1-\theta}{\theta_1}\sin\gamma\right)\right\}$$
$$=\frac{R^3}{\varepsilon\theta}Q\cdot\left(\frac{\sigma+\sigma_1}{2}-\frac{\sigma_2{}^2}{\sigma}\right),$$

where σ_1 also has the value given on p. 12. Hence employing the symbol $\varkappa$ with the signification assigned on p. 15, we have

$$(33)\qquad Q=\frac{2\lambda a\varepsilon\theta}{\varkappa R^3}=\frac{2\lambda\varepsilon\theta\sin\alpha}{\varkappa R^2}.$$

Having thus found Q, the moment Na is found by (29), which gives

$$(34)\qquad Na=-\left(\frac{\sigma_2}{\sigma}-\cos\alpha\right)QR.$$

We can now compute the moment of flexure at the different points of the arch by the equation

$$M=Na+Qy$$

α (in arc) $= 0.3947912$	γ (in arc) $= 0.3262709$
$\sigma = 0.3719511$	$\log \sigma = 9.5704859$
$\sigma_1 = 0.3378877$	$\log \sigma_1 = 9.5287724$
$\sigma_2 = 0.3632479$	$\log \sigma_2 = 9.5602031$
$\sigma_3 = 0\ 9311328$	$\log \sigma_3 = 9.9690116$
$\sigma_4 = 0.06643108$	$\log \sigma_4 = 8.8223713$
$\sigma_5 = 0.008150965$	$\log \sigma_5 = 7.9112090$
$\sigma_6 = 0.2692211$	$\log \sigma_6 = 9.4301091$
$x = 0.0003422$	$x_2 = 0.2441503$
$x_1 = 0.9757763$	$x_3 = 0.000165623$

The computations being made for points of the arch the horizontal distance of which is $\frac{1}{16}$ of the whole span, the values of φ for these points are found by the formulæ

$$\sin \varphi = (1 - m) \sin \alpha$$

by taking $m = 0$, $\frac{1}{8}$, $\frac{2}{8}$, &c., up to $m = 2$. The values of φ for the values of $m = \frac{9}{8}$, $\frac{10}{8}$, &c., being merely the negatives of those for $\frac{7}{8}$, $\frac{6}{8}$, &c., it suffices to compute the positive values of φ. These are given in the following table:

$8m$	φ	ϕ (in arc)	$\log \sin \phi$	$\log \cos \phi$
0	22°37′ 11″.52	0.3947912	9.5850267	9.9652379
1	19 39 58 .03	.3432386	9.5270348	9.9738986
2	16 45 57 .16	.2926198	9.4600880	9.9811349
3	13 54 33 .27	.2427621	9.3809067	9.9870751
4	11 5 14 .96	.1935132	9.2839967	9.9918171
5	8 17 33 .96	.1447361	9.1590580	9.9954351
6	5 31 3 .85	.0963027	8.9829667	9.9979830
7	2 45 20 .40	.0480955	8.6819367	9.9994975
8	0 0 0 .00	.0000000	inf. neg.	0.0000000

The computations for the effect of the movable load are made for the cases in which the load covers $\frac{1}{8}$, $\frac{2}{8}$, $\frac{3}{8}$, &c., of the whole span, reckoning from one extremity. The values of β corresponding to the points at which the load terminates are found by the formula

$$\sin \beta = (1 - n) \sin \alpha,$$

taking $n = \frac{1}{4}$, $\frac{2}{4}$, &c., up to $\frac{8}{4}$. With these are then computed the auxiliaries

$$\mu = \beta + 2\beta \sin^2 \beta + 3 \sin \beta \cos \beta,$$

$$\xi = \frac{\cos \beta}{2} + \frac{\beta \sin \beta}{2} - \frac{\cos^3 \beta}{6},$$

$$\nu = \sin \beta + \frac{\cos^3 \beta}{6}.$$

NUMERICAL RESULTS OF THE PRECEDING FORMULÆ.

The computations have been made only for the central arch because this is the largest. For this we have

$$R = 673.5 \text{ ft.} \qquad \alpha = 22^\circ\ 37'\ 11''.52$$
$$\delta = 8\ \text{“} \qquad a = 259.0385 \text{ ft.}$$

The stronger end pieces extend to $\frac{1}{12}$ the span, and hence

$$\sin\gamma = \tfrac{5}{6}\sin\alpha, \qquad \gamma = 18^\circ\ 41'\ 48''.18.$$

The sectional area of a member is

For centre piece, $\omega = 80$ sq. in. $= \frac{5}{9}$ sq. ft.
For end pieces, $\omega_1 = 120$ sq. in. $= \frac{5}{6}$ sq. ft.

The modulus of elasticity of steel has been taken as

$$\epsilon = 2232000 \text{ tons per sq. foot.}$$

Hence

$$\epsilon\theta = \frac{\epsilon\omega\delta^2}{2} = 1240000 \times 32 = 39680000,$$

$$\epsilon\theta_1 = \frac{\epsilon\omega_1\delta^2}{2} = 1860000 \times 32 = 59520000.$$

Also,

$$\frac{\theta}{\theta_1} = \frac{2}{3}, \qquad \frac{\theta_1 - \theta}{\theta_1} = \frac{1}{3}.$$

We first compute the constants σ, σ_1, &c., by the formulæ

$$\sigma = \frac{\theta}{\theta_1}\alpha + \frac{\theta_1-\theta}{\theta_1}\gamma, \qquad \sigma_4 = \frac{\theta}{\theta_1}\cdot\frac{\sin^2\alpha}{2} + \frac{\theta_1-\theta}{\theta_1}\cdot\frac{\sin^2\gamma}{2},$$

$$\sigma_1 = \frac{\theta}{\theta_1}\sin\alpha\cos\alpha + \frac{\theta_1-\theta}{\theta_1}\sin\gamma\cos\gamma, \qquad \sigma_5 = \frac{\theta}{\theta_1}\cdot\frac{\sin^3\alpha}{6} + \frac{\theta_1-\theta}{\theta_1}\cdot\frac{\sin^3\gamma}{6},$$

$$\sigma_2 = \frac{\theta}{\theta_1}\sin\alpha + \frac{\theta_1-\theta}{\theta_1}\sin\gamma \qquad \sigma_6 = \frac{\theta}{\theta_1}\cdot\frac{\cos^3\alpha}{3} + \frac{\theta_1-\theta}{\theta_1}\cdot\frac{\cos^3\gamma}{3}.$$

$$\sigma_3 = \frac{\theta}{\theta_1}\cos\alpha + \frac{\theta_1-\theta}{\theta_1}\cos\gamma,$$

$$\varkappa = \sigma + \sigma_1 - \frac{2\sigma_2^2}{\sigma}, \qquad \varkappa_2 = \frac{\sigma_2}{4\sigma},$$

$$\varkappa_1 = \frac{\sigma_2\sigma_3}{\sigma} + \sigma_4, \qquad \varkappa_3 = \varkappa_2(\sigma - \sigma_1) - \sigma_5.$$

RESISTANCES OF THE PIERS FOR MOVABLE LOADS.

(q = 0.8 ton per running foot.)

$8n$	P	Q	Na
	Tons	Tons.	Foot-tons.
2	51.2	7.5	+ 1194
4	98.6	51.2	+ 3089
6	138.7	141.3	+ 3994
8	169.6	260.8	+ 3543
10	190.6	380.4	+ 2191
12	202.2	470.4	+ 739
14	206.6	514.1	— 107
16	207.2	521.6	— 272

RESISTANCES OF THE PIERS FOR PERMANENT LOAD.

(q' = 1 ton per running foot.)

P	Q	Na
259.0	652.0	—340

We now proceed to compute the moments of flexure resulting from the movable loads for the several points of the arch, and for each condition of the load, by the formulæ:

FOR LOADED PART,

$$M = Na - Pam + \frac{qa^2}{2}m^2 + Qy;$$

FOR UNLOADED PART,

$$M = Na - Pam + qna^2\left(m - \frac{n}{2}\right) + Qy.$$

The values of y, computed by the formula $y = R(\cos\varphi - \cos\alpha)$, are as follows:

$8m$		log y	y in feet.
0	16	inf. neg.	0.00000
1	15	1.0976824	12.52225
2	14	1.3650799	23.17821
3	13	1.5059519	32.05914
4	12	1.5936914	39.23660
5	11	1.6509452	44.76558
6	10	1.6874133	48.68703
7	9	1.7078162	51.02890
8	8	1.7143943	51.80770

MOMENTS OF FLEXURE ($=M$) PRODUCED BY MOVABLE LOADS.

($q=0.8$ ton.)

$8m$	$n=\frac{1}{4}$	$n=\frac{2}{4}$	$n=\frac{3}{4}$	$n=\frac{4}{4}$	$n=\frac{5}{4}$	$n=\frac{6}{4}$	$n=\frac{7}{4}$	$n=\frac{8}{4}$
0	+1194	+3089	+3094	+3542	+2190	+ 739	— 107	—272
1	+ 50	+ 957	+1688	+1732	+1198	+ 496	+ 54	— 37
2	— 270	— 431	— 42	+ 276	+ 337	— 217	+ 97	+ 66
3	— 182	—1071	—1174	— 809	— 355	— 50	+ 74	+ 90
4	— 109	— 959	—1723	—1485	— 858	— 286	+ 11	+ 62
5	— 47	— 513	—1658	—1758	—1146	— 450	— 57	+ 16
6	+ 2	— 149	— 978	—1617	—1215	— 546	— 123	— 42
7	+ 40	+ 134	— 112	—1047	—1045	— 541	— 158	— 80
8	+ 66	+ 338	+ 534	— 45	— 626	— 428	— 149	— 92
9	+ 79	+ 461	+ 965	+ 968	+ 36	+ 213	— 118	— 80
10	+ 82	+ 504	+1174	+1576	+ 946	+ 107	— 43	— 42
11	+ 72	+ 466	+1162	+1774	+1667	+ 529	+ 64	+ 16
12	+ 50	+ 346	+ 919	+1546	+1785	+1022	+ 171	+ 62
13	+ 17	+ 142	+ 446	+8912	+1191	+1158	+ 274	+ 90
14	— 30	— 150	— 270	— 210	+ 107	+ 497	+ 338	+ 66
15	— 90	— 533	—1235	—1770	—1726	— 994	— 87	— 37
16	— 165	—1010	—2462	—3814	—4266	—3361	—1466	—272

MOMENTS OF FLEXURE PRODUCED BY PERMANENT LOAD.

($q=1$ ton.)

$8m$	0	1	2	3	4	5	6	7	8
M	—340	—46	83	113	77	20	—53	—100	—115

We next find the strains in the direction of the arch for the several points by the formula

$$S' = Q\cos\varphi + V\sin\varphi,$$

employing for V the values $V = P - qma$ for loaded part,

$$V = P - qna \text{ for unloaded part.}$$

STRAINS IN DIRECTION OF ARCH ($=S'$) PRODUCED BY MOVABLE LOADS.

($q = 0.8$ ton.)

Sm	$n=\frac{1}{4}$	$n=\frac{2}{4}$	$n=\frac{3}{4}$	$n=\frac{4}{4}$	$n=\frac{5}{4}$	$n=\frac{6}{4}$	$n=\frac{7}{4}$	$n=\frac{8}{4}$
0	26.63	85.20	183.77	306.00	424.38	511.99	554.04	561.22
1	15.59	72.70	171.01	293.98	413.58	502.30	544.94	552.22
2	7.02	62.54	160.34	283.71	404.21	493.78	536.90	544.30
3	7.14	54.74	151.80	275.26	396.33	486.53	530.02	537.47
4	7.26	49.30	145.39	268.65	389.98	480.58	524.34	531.84
5	7.35	49.96	141.13	263.88	385.18	475.98	519.86	527.40
6	7.42	50.51	139.03	260.98	381.97	472.72	516.65	524.21
7	7.49	50.93	140.31	259.95	380.36	470.86	514.74	522.28
8	7.52	51.22	141.27	260.82	380.35	470.40	514.11	521.64
9	7.54	51.42	141.91	262.33	381.97	471.35	514.79	522.28
10	7.55	51.47	142.22	263.22	385.18	473.69	516.77	524.21
11	7.53	51.42	142.21	263.52	386.26	477.44	519.85	527.40
12	7.49	51.24	141.85	263.18	386.43	482.52	524.57	531.84
13	7.45	50.93	141.14	262.21	384.86	482.73	530.31	537.47
14	7.38	50.50	140.07	260.58	383.94	481.74	537.26	544.30
15	7.28	49.93	138.64	258.26	381.21	479.52	536.63	552.22
16	7.18	49.22	136.82	255.22	377.42	476.00	534.56	561.22

STRAINS IN DIRECTION OF ARCH PRODUCED BY PERMANENT LOAD.

($q = 1$ ton.)

m	0	1	2	3	4	5	6	7	8
S'	701.52	690.28	680.37	671.84	664.80	659.25	655.26	652.85	652.05

The shearing strains follow, computed by the formula

$$S'' = Q \sin \varphi - V \cos \varphi,$$

employing the same values of V as before.

SHEARING STRAINS PRODUCED BY MOVABLE LOADS.

($q = 0.8$ ton.)

$8m$	$n=\frac{1}{4}$	$n=\frac{2}{4}$	$n=\frac{3}{4}$	$n=\frac{4}{4}$	$n=\frac{5}{4}$	$n=\frac{6}{4}$	$n=\frac{7}{4}$	$n=\frac{8}{4}$
0	—44.36	—71.28	—73.76	—56.24	—29.68	— 5.68	+ 7.04	+ 9.34
1	—21.29	—51.20	—58.72	—47.60	—27.04	— 7.68	+ 2.88	+ 4.80
2	+ 2.76	—30.00	—42.48	—37.52	—23.20	— 8.32	+ 0.08	+ 1.67
3	+ 2.41	— 7.92	—25.28	—26.48	—18.08	— 7.76	— 1.52	—0.33
4	+ 2.05	+ 14.80	— 7.28	—14.64	—12.16	— 6.24	— 2.16	—1.37
5	+ 1.70	+ 12.40	+ 11.28	— 2.08	— 5.52	— 4.08	— 2.16	—1.66
6	+ 1.34	+ 10.00	+ 30.24	+ 10.96	+ 1.60	— 1.36	— 1.52	—1.41
7	+ 0.98	+ 7.52	+ 23.44	+ 24.24	+ 9.12	+ 1.76	— 0.56	—0.79
8	+ 0.62	+ 5.04	+ 16.72	+ 37.60	+ 16.64	+ 5.04	+ 0.56	0.00
9	+ 0.26	+ 2.56	+ 9.92	+ 25.04	+ 24.24	+ 8.24	+ 1.76	+ 0.79
10	— 0.10	+ 0.08	+ 3.04	+ 12.32	+ 31.60	+ 11.36	+ 2.72	+ 1.41
11	— 0.48	— 2.32	— 3.92	— 0.40	+ 12.88	+ 14.08	+ 3.36	+ 1.66
12	— 0.85	— 4.88	—10.80	—13.28	— 5.84	+ 16.16	+ 3.44	+ 1.37
13	— 1.21	— 7.44	—17.76	—26.24	—24.96	— 7.60	+ 2.72	+ 0.33
14	— 1.58	—10.00	—24.80	—39.20	—44.16	—31.68	+ 1.92	—1.67
15	— 1.95	—12.48	—31.76	—52.40	—63.52	—56.00	—26.08	—4.80
16	— 2.33	—14.96	—38.96	—65.60	—83.04	—80.64	—53.76	—9.34

SHEARING STRAINS PRODUCED BY PERMANENT LOAD.

($q = 1$ ton.)

$8m$	0	1	2	3	4	5	6	7	8
S''	11.67	6.00	2.09	—0.41	—1.71	—2.08	—1.76	—0.99	0.00

The effect of a change of temperature in the arch is computed by the following formulæ:

For a change of temperature of 80° F. we take

$$\lambda = 0.000527;$$

and the resistances of the piers caused by this change are found by the formulæ

$$Q = \frac{2\lambda\varepsilon\theta \sin\alpha}{\varkappa R_{1,2}^2}, \qquad Na = -\left(\frac{\sigma_2}{\sigma} - \cos\alpha\right) QR;$$

the moments of flexure by the formula

$$M = Na + Qy;$$

the strains in the direction of the arch by

$$S' = Q\cos\varphi;$$

the shearing strains by

$$S'' = Q\sin\varphi.$$

EFFECTS OF CHANGE OF 80° F. IN TEMPERATURE ON THE ARCH.

Q = 103.7 tons = horizontal force at piers.
Na = —3739 tons = moment at piers.

$8m$	Moment of flexure = M	Strains in direction of arch = S'	Shearing strains = S''
	Foot tons	Tons.	Tons.
0	—3739	96	40
1	—2440	98	35
2	—1335	99	30
3	— 414	101	25
4	+ 331	102	20
5	+ 905	103	15
6	+1311	103	10
7	+1554	104	5
8	+1635	104	0

The deflections have been computed for the cases of most interest, namely, for $n = \frac{3}{4}$, $n = 1$, and $n = \frac{5}{4}$; as a load covering about one half the span produces the greatest deflections.

The first step is to compute the constants A, B, C, etc. Their values found by the formulæ on pp. 19 and 20, are as follows:*

VALUES OF CONSTANTS.

	$n=\frac{3}{4}$	$n=1$	$n=\frac{5}{4}$
A	— 68.46599	— 37.39385	— 16.53583
A_1	— 68.38344	— 37.31483	— 16.48398
B_1	— 16.49586	— 37.31483	— 68.37156
B	— 16.55112	— 37.39738	— 68.45774
C	— 68.45513	— 37.38182	— 16.52660
C_1	— 68.37811	— 37.30817	— 16.47867
D_1	— 16.49058	— 37.30817	— 68.36620
D	— 16.54218	— 37.38521	— 68.44710
E	+ 41.59079	— 81.31384	—201.71064
E_1	+ 41.61988	— 81.28565	—201.69250
F_1	—140.46900	—260.88565	—383.78137
F	—140.44978	—260.85612	—383.75065

* It may be well to remark that in the computation of the deflections logarithms of six or seven decimal places should be used, although the quantities P and Q which enter into the formulæ have not been determined accurately to more than three or four figures; the deflections result from very small differences in certain functions of the angular quantities. The results will be sensibly correct even with imperfect values of

If now in the formulæ for $x_1 - x$ and $y_1 - y$ (p. 14), we multiply by the factors $\frac{R^3}{\epsilon\theta_1}$ and $\frac{R^3}{\epsilon\theta}$, the results may be written thus:

$$x_1 - x = -b(\sin\varphi - \varphi\cos\varphi) + c\sin^2\varphi + d(\varphi - \sin\varphi\cos\varphi) - e\sin^3\varphi + f\cos\varphi + g,$$

$$y_1 - y = b(\cos\varphi + \varphi\sin\varphi) - c(\varphi + \sin\varphi\cos\varphi) - d\sin^2\varphi - e\cos^3\varphi + f\sin\varphi + h,$$

the coefficients b, c, d, e, f, g, h, receiving their proper values for the several cases. These values are given in the following table:

COEFFICIENTS.

		Loaded end-piece.	Loaded part of centre-piece.	Unloaded part of centre piece.	Unloaded end-piece.
$n=\frac{3}{4}$	$\log b$	1.2266803	1.4027715	3.0340377	2.8579465
	$\log c$	2.2451421	2.4212333	1.8084806	1.6323894
	$\log d$	2.5593698	2.7354610	2.7354610	2.5593698
	$\log e$	2.3625963	2.5386875	*	* .....
	$\log f$	2.5458283	2.7213920	2.1038160	1.9291773
	g	—351.3629	—526.4513	— 126.9630	— 84.9068
	h	+ 213.475	+ 320.436	—1081.488	—720.893
$n=1$	$\log b$	2.8113577	2.9874489	3.3028903	3.1267991
	$\log c$	1.9846327	2.1607239	1.1607239	3.9846327
	$\log d$	2.8256608	3.0017520	3.0017520	2.8256608
	$\log e$	2.3625963	2.5386875	*	* .....
	$\log f$	2.2831501	2.4583225	2.4583225	2.2831911
	g	—191.8714	—287.2401	— 287.2401	— 191.8888
	h	—417.364	—625.828	—2008.590	—1338.909
$n=\frac{5}{4}$	$\log b$	3.1023182	3.2784094	3.4705198	3.2944286
	$\log c$	1.6320577	1.8081489	2.4211528	2.2450616
	$\log d$	2.9895147	3.1656059	3.1656059	2.9895147
	$\log e$	2.3625963	2.5386875	*	* .....
	$\log f$	1.9287759	2.1035032	2.7213166	2.5457725
	g	— 84.8269	— 126.8713	— 526.3596	— 351.3216
	h	—1035.330	—1552.855	—2954.779	—1969.695

* For unloaded part of arch $e = o$.

P and Q, *provided the* SAME *values* of P and Q are employed in finding the constants A, B, C, etc., which are afterwards used in the formulæ for $x_1 - x$ and $y_1 - y$. In our computation, we have taken

for $n=\frac{3}{4}$, $P = 138.71000$ $\log Q = 2.1500499$,
" $n=1$, $P = 169.62000$ $\log Q = 2.4163409$,
" $n=\frac{5}{4}$, $P = 190.53000$ $\log Q = 2.5801948$.

The deflections are computed by each $\frac{1}{16}$ of the span, that is, for values of $x = ma$, denoted by $m = 0$, $\frac{1}{8}$, $\frac{2}{8}$, etc., to $m = 2$. It is convenient to prepare such a table as the following for the logarithms of the special functions $\sin\varphi - \varphi\cos\varphi$, etc.

LOGARITHMS OF FUNCTIONS.

$8m$	Log $\sin\phi - \varphi\cos\varphi$	Log $\phi - \sin\varphi\cos\phi$	Log $\cos\varphi + \varphi\sin\varphi$	Log $\phi + \sin\varphi\cos\phi$
0	8.305197	8.599464	0.0313761	9.874958
1	8.124543	8.420458	0.0241501	9.819641
2	7.918062	8.215378	0.0178265	9.754975
3	7.675851	7.974332	0.0124298	9.677697
4	7.381386	7.680789	0.0079818	9.582326
5	7.003504	7.303930	0.0045014	9.458576
6	6.473487	6.774079	0.0020046	9.283326
7	5.569374	5.870404	0.0005017	8.982799
8	inf. neg.	inf. neg.	0.0000000	inf. neg.

With the aid of these tables and those of $\log\sin\varphi$ and $\log\cos\varphi$, previously given, the following deflections have been found:

DEFLECTIONS OF THE ARCH PRODUCED BY MOVABLE LOADS.

($q = 0.8$ ton.)

	$n = \frac{3}{4}$		$n = 1$		$n = \frac{5}{4}$	
$8m$	$x_1 - x$	$y_1 - y$	$x_1 - x$	$y_1 - y$	$x_1 - x$	$y_1 - y$
	inches.	inches.	inches.	inches.	inches	inches.
0	0.00	0.00	0.00	0.00	0.00	0.00
1	+ 0.13	— 0.33	+ 0.13	— 0.36	+ 0.11	— 0.23
2	+ 0.39	— 1.18	+ 0.37	— 1.07	+ 0.24	— 0.73
3	+ 0.60	— 1.93	+ 0.60	— 1.92	+ 0.41	— 1.36
4	+ 0.69	— 2.29	+ 0.74	— 2.52	+ 0.52	— 1.87
5	+ 0.66	— 2.17	+ 0.80	— 2.69	+ 0.58	— 2.10
6	+ 0.58	— 1.51	+ 0.71	— 2.22	+ 0.54	— 1.98
7	+ 0.51	— 0.52	+ 0.65	— 1.32	+ 0.51	— 1.44
8	+ 0.49	+ 0.48	+ 0.61	— 0.06	+ 0.49	— 0.61
9	+ 0.51	+ 1.36	+ 0.64	+ 1.20	+ 0.51	+ 0.41
10	+ 0.55	+ 1.91	+ 0.71	+ 2.17	+ 0.59	+ 1.43
11	+ 0.56	+ 2.05	+ 0.75	+ 2.63	+ 0.65	+ 2.12
12	+ 0.53	+ 1.88	+ 0.75	+ 2.56	+ 0.70	+ 2.33
13	+ 0.47	+ 1.47	+ 0.62	+ 2.00	+ 0.61	+ 1.98
14	+ 0.27	+ 0.84	+ 0.39	+ 1.14	+ 0.41	+ 1.22
15	+ 0.09	+ 0.25	+ 0.13	+ 0.35	+ 0.12	+ 0.37
16	0.00	0.00	0.00	0.00	0.00	0.00

Explanation of the Diagrams.

The preceding numerical results are graphically exhibited in the diagrams of Plates I, II, and III, in order to facilitate their examination and discussion.

The horizontal length of each diagram represents the span ($2a$) of one rib, and is divided into sixteen equal parts ($=\frac{a}{8}$ each) by vertical lines, which correspond therefore to the successive values of m in our tables, beginning on the left with $m=0$, and ending on the right with $m=2$. Moments and strains at the several points of the span are represented by vertical distances at these points, on any arbitrary scale; the values in the tables being laid off above and below the zero line according to their algebraic signs. The extremities of the distances thus laid off are then joined, and there result curves which exhibit to the eye the law which the strains follow, and enable us to deduce readily the maximum total strains by means of the curves which *envelop* all the other curves; they also facilitate the determination of the sectional areas of the ribs necessary to withstand these strains.

PLATE I. *Diagram* 1.—In this diagram the algebraic sums of the moments (M) of flexure produced by both movable and permanent load, as given in the tables on p. 33, are laid down as ordinates to the values of m as abscissas, on the scale indicated on the *left* of the diagram. Positive values of M are measured upwards, negative values downwards, from the line 0—0. The numbers 0, $\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$.....$\frac{7}{8}$, 1, by which the curves are marked, indicate the extent of the movable load in accordance with the figures below the diagram; and these numbers correspond to the values of $n=0$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$.....$\frac{7}{4}$, 2, respectively, of our tables, since the numbers in the diagrams refer to the whole span, and the values of n to the half span. Instead of forming an additional table and a corresponding diagram for the strains resulting from the moments of flexure, we have only to consider the same diagram as exhibiting either moments or strains by applying to it two different scales; for the strains are deduced from the moments by dividing by the constant 8 (=distance of centres of members). The scale on the right is therefore one eighth of the scale on the left, and represents strains, compressive or tensile, according to sign.

In Diagram 2, Plate I, the forces (S') in the direction of the arch are first laid down from the tables on p. 34, and the resulting curves are distinguished by the use of the species of line (— · — · — · —). The force S' being equally distributed between the two members, the distances of the curves (— · — · — · —) from the axis 0—0 represent one half the sum of the strains S' given in the tables for movable and for permanent loads,

on the scale indicated on the left of the diagram. These strains are all compressive. The strains resulting from the moments of flexure are tensile in one member and compressive in the other, and the total strains in the members are obtained by laying off from the curves (— · — · — · —) the strains taken from Diagram 1 (by scale on the right of that diagram), *upwards* for *tension*, and *downwards* for *compression*. We thus obtain two new curves for each value of n, one of which (a broken line — — —) shows the total strains by movable and permanent load in the upper member, and the other (a full line ———) shows the same for the lower member. Enveloping curves are now drawn touching all these curves (distinguished by heavier full and broken lines), which exhibit the maxima and minima of compression and tension. The lower enveloping curve to the curves belonging to one member exhibits the maximum compression, and the upper enveloping curve to the same curves exhibits the maximum tension (minimum compression) for the different points of this member, when a load covers the bridge progressively from the left to the right. For a load covering the bridge progressively from the right to the left, these curves will all be reversed. Therefore curves which represent the maxima of strains for *every* kind of load (loads from *both* directions) must be symmetrical with respect to the centre line of the span; they will be obtained by drawing the enveloping curves in both positions (direct and reversed) and then taking for the final curve of maxima those portions of the compound curve thus formed which lie farthest from the axis. Such are the heavy lines (full and broken) of Diagram 2. We see that by the effect of load we get tension only for a very short distance from the abutments. The heavy dotted lines show the minima of compressive strain in one member.

The scale on the right side of Diagram 2 gives the amount of efficient sectional area required to withstand with safety the strains indicated on the left side; allowing for compression 12.5 tons, and for tension 10 tons, per square inch. The limit of elasticity of our steel is not less than 25 tons per square inch. Recent experiments indeed indicate that the limit is considerably higher, probably over 30 tons per square inch. By employing 12.5 tons, we are therefore very far within the limits of safety, and it may even be expedient to further reduce the weight, and consequently also the cost of the structure, by taking smaller sectional areas, say such as correspond to at least 15 tons per square inch (for comp.).

PLATE II. *Diagram* 1.—This is constructed from the strains in the direction of the arch and the moments of flexure caused by a decrease

crease of temperature of 80° F., as given in the tables on p 36. The

od of construction is precisely the same as that used in Diagram 2

ate I.

Diagram 2 of Plate II the light lines are mere copies of the heavy

lines of Diagram 2, Plate I, and represent the maxima of compression and tension (or minima of compression) caused by load only without the effect of temperature. The heavy lines represent the combined effect of load and temperature, and are obtained from the curves for load by adding to the ordinates of these curves in both directions (upward and downward) the corresponding ordinates of Diagram 1. Here also the scale on the right side indicates the sectional area required in a member to withstand the strains indicated by the scale on the left, allowing as before for compression 12.5 tons and for tension 10 tons to the square inch. We see that the highest total strains in one member occur at the abutments, and that they decrease rapidly with the distance from them. For more than three fourths of the span, in the central part of it, there is not much variation in the maximum strains; and even with 12.5 tons per square inch, we see that a sectional area of 72 square inches in a member would suffice for this part of the bridge. We also observe that by the combined effect of load and temperature we get tensile strain only very near the abutment.

PLATE III shows the strains in the braces. We suppose that the members resist strains only in the direction of the arch, so that the total strain normal to the arch must be taken up by the braces. This is not quite correct, but very nearly so, as the stiffness of a single member against bending is very small in comparison with the stiffness of the whole rib. Our supposition gives the strains for the braces a little too great, and can therefore only increase and not diminish the safety of our calculations.

In Diagram 1 of Plate III the algebraic sums of the shearing strains (S'') by movable and permanent load, as given in the tables on p. 35, are laid down as ordinates, upon the scale indicated on the left side, to the values of m as abscissas. For the nine cases of load we get the nine lines marked by the numbers 0, $\frac{1}{8}$, $\frac{1}{4}$ ..... $\frac{7}{8}$, 1 (corresponding respectively, to the successive values of $\frac{n}{2}$), which numbers express the portion of the span that is loaded, according to the figures between Diagrams 1 and 2. The heavy full lines are the upper and lower enveloping curves, and give the maxima and minima of shearing strain occurring in the different sections of the rib, when a load covers the bridge progressively from the left to the right. The broken lines represent the same for a load coming from the right, and are the same as the full lines reversed, by changing right to left and left to right, and at the same time + to — and — to +.

We distinguish between the two kinds of braces a and b, as indicated in Diagram 2 of Plate III. The angle of the braces with the centre line of the arch is $\pm 60°$. In any normal section through the arch we only

find one kind of braces efficient; so that, if S'' is the shearing strain, the strain in the braces of this section will be

$$+\frac{S''}{\sin 60^\circ} = +1.155\ S''$$ if they are of the kind a,

and $$-\frac{S''}{\sin 60^\circ} = -1.155\ S''$$ if they are of the kind b.

The sign + signifies compressive and the sign — tensile strain. Therefore, a

$$\left[\begin{matrix}\text{positive}\\ \text{negative}\end{matrix}\right]$$ value of S'' causes $$\left[\begin{matrix}\text{compression}\\ \text{tension}\end{matrix}\right]$$ in a,

and a $$\left[\begin{matrix}\text{positive}\\ \text{negative}\end{matrix}\right]$$ value of S'' causes $$\left[\begin{matrix}\text{tension}\\ \text{compression}\end{matrix}\right]$$ in b.

The strains in the braces of a section of the rib for the different values of S'' are given by the scale on the right side of Diagram 1, Plate III, which contains the number on the left multiplied by 1.155.

Diagram 3, Plate III, is constructed from the shearing strains S'' caused by a change of temperature of 80°, according to the table on p. 36, in the same manner as Diagram 1 from the shearing strains by load.

In Diagram 4, Plate III., the light lines show the possible maximum of strain by the load in the braces of a section of the rib (obtained from Diagram 1 by the scale on the right). The full lines give the maxima of compression in braces a or of tension in braces b, and the broken lines give the maxima of tension in braces a or of compression in braces b. The heavy lines which show the possible maxima of strain by load and temperature together are obtained from the corresponding light ones by adding the strains by change of temperature taken from Diagram 3, using the scale on the right. The scale on the right of Diagram 4 gives the efficient sectional area required in the braces for the strains indicated by the scale on the left, allowing 10 tons per square inch for either compression or tension.

The shaded line, composed of the extreme parts of the heavy lines, gives the greatest absolute strains for the different sections of a rib, and therefore also the maximum of required sectional area.

Effect of Compression by the Strains in the Direction of the Arch.

The effect of these strains has been omitted in the calculation of the resistances exerted by the abutments. We shall now investigate the error produced by this omission.

By the strains in the direction of arch the elements of the arch are compressed in their original direction, and therefore shortened without bending. Suppose the span $2a$ of the original arch is reduced in this way to $2a_1$. The distance between the abutments remains unchanged, and therefore, to find the real resistances of the abutments, we have to add to those found in the computation above such forces as will cause an increase of span of $2(a-a_1)$. As the load remains unchanged, the effect of a compression in the direction of arch is the same as that of a decrease of temperature, for which the coefficient of contraction is determined by the equation

$$\lambda = \frac{a-a_1}{a}.$$

For a certain load we can avoid these additional strains by constructing the ribs so much longer than the distance between the abutments, that, when shortened by the compression, they will just fit between them.

From Diagram 2, Plate I, we see that the strains in the direction of arch are very nearly equal through the whole span, and vary for different loads from about 350 to about 600 tons in one member. If we assume, as in the former calculations, the sectional area of one member in the end-pieces ($\frac{1}{12}$ of span each) = 120 square inches, and in centre-piece = 80 square inches, and the modulus of elasticity of cast steel = 15500 tons per square inch, the decrease of span produced by a strain of 1 ton in one member will be

$$2(a-a_1) = \frac{2a}{6}\cdot\frac{1}{120\times 15500} + \frac{10a}{6}\cdot\frac{1}{80\times 15500} = 2a\times\frac{17}{22320000},$$

and now $$\lambda = \frac{a-a_1}{a} = \frac{17}{22320000} = 0.000000762.$$

The coefficient of contraction of cast steel for a decrease of temperature of 80° F. is 0.000527; therefore the effect of a compressive strain of 1 ton in one member of a rib will be equal to the effect of a decrease in temperature of

$$\frac{80}{0.000527}\times 0.000000762 = 0°.116 \text{ F.}$$

For our bridge, where the strains in direction of arch vary from 350 tons to 600 tons per member, we would have to add to the strains obtained by the foregoing calculations the strains caused by decreases in temperture of $350\times 0.116 = 40°.6$ F. up to $600\times 0.116 = 69°.6$ F.

If we had always the same load, we could wholly avoid these additional strains in the above indicated manner. For our variable loads we can at least reduce them very much by constructing the ribs of such

length that they will be brought to the span $2a$ by a mean compressive strain of $\frac{350 + 600}{2} = 475$ tons per member. This will act like an increase of temperature of $475 \times 0.116 = 55°.1$ F. It will wholly destroy the additional strains for a load on $\frac{1}{2}$ of a span, and reduce them for all other loads to values not exceeding the effects of a change in temperature (increase or decrease) of 14°. 5 F. The heaviest total strains, and therefore also the maxima of required sectional areas are obtained for a movable load on $\frac{3}{8}$, $\frac{1}{2}$ and $\frac{5}{8}$ of span. For these cases the strains in direction of arch will not differ from 475 tons per member more than about 60 tons, and the additional strains will not surpass the effect of a change in temperature of $60 \times 0.116 = 7°$ F. This effect is very small, and can be allowed for in construction by choosing the sectional areas a little larger than those given by the diagrams. But even if we neglect it altogether, our calculation will still be safe for all degrees of temperature within $(80° - 7°) = 73°$ F. above and below the assumed normal temperature of 60° F.

The ratio by which we have to multiply all dimensions for the construction is given by

$$1 + \frac{0.000527 \times 55.1}{80} = 1.000363.$$

The length by which the span must be constructed larger than the distance between the abutments, is

$$2a \,.\, 0.000363 = 0.1864 \text{ feet} = 2.256 \text{ inches}.$$

EFFECT OF WIND ON THE SUPERSTRUCTURE.

The superstructure contains three systems of bracing against horizontal forces,—one between the steel arches, one between the beams of the lower roadway, and one formed by the timbers of the upper roadway. Each of these systems forms a truss against the wind.

The first two consist of longitudinal members and diagonal braces between them. If we denote by

q the horizontal force of wind per linear foot taken up by one truss,
$2a$ the span of truss,
x the distance of the centre of a panel from one abutment,
α the angle between members and diagonals;

and if further (as in the calculation of the steel arches) we suppose that all strain normal to the truss is taken up by the braces, we get for the strain S of all the braces in one panel

$$S = q \frac{a - x}{\sin \alpha}$$

For $x = a$, or for the centre of span, we have $S = 0$, and it increases proportionally to the distance from the centre, and attains its greatest value, $q \frac{a}{\sin \alpha}$, for the abutment.

The force of a tornado, tearing up trees, etc., is about 50 pounds to a square foot; and we suppose that the superstructure will not oppose to the wind more than 40 square feet per foot of length. This gives for the whole superstructure

$$q = 2000 \text{ lbs.} = 1 \text{ ton.}$$

For safety, we will suppose that the bracing between the arches should be able to stand the whole pressure of wind. For this we have

$$\alpha = 51°, \text{ and } \frac{1}{\sin \alpha} = 1.287, \text{ say } = 1.3,$$
$$a = 259' \text{ (half span of centre line of arch),}$$

and now for S the equation,

$$S = 1.3\,(259 - x) \text{ tons},$$

which gives

for $x =$	0	50′	100′	150′	200′	259′
$S =$	336.7 tons	271.7 tons	206.7 tons	141.7 tons	76.6 tons	0.

Between the two inner ribs each panel has two braces (one between upper members, and one between lower members). Between the inner and outside ribs the bracing cannot extend farther than about 100 feet from each abutment. The bracing of the outside spaces is only between the upper members. The total number of braces in one panel is therefore

$$4 \text{ from } x = 0 \text{ to } x = 100',$$
$$\text{and } 2 \text{ from } x = 100' \text{ to } x = a.$$

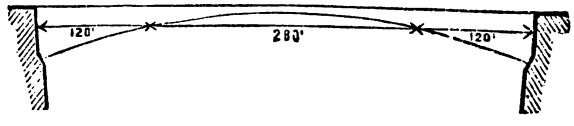
If we make the braces of cast steel, and allow 12.5 tons strain per square inch (a little more than for the braces in the arches, because such a tornado will occur but very seldom), we get the following table, in which

s = the strain on one brace,
A = the sectional area required in one brace.

x in feet.	0	50′	100′		150′	200′	259′
			4 braces	2 braces			
s in tons	84.2	67.9	51.7	103.4	70.8	38.3	0
A in square inches	6.7	5.4	4.1	8.3	5.7	3.1	0

The truss formed by bracing between the beams carrying the railroad tracks is supported at the piers, and at a distance of about 120 feet from

Fig. 4.

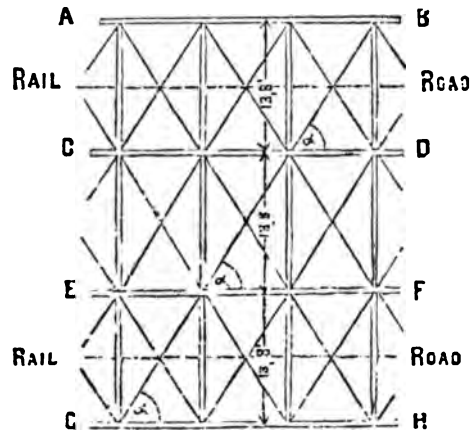


the piers by the arches, so that the length of this truss is divided into one centre span of about 280′, and two side spans of 120′ each.

The force of wind to be taken up by this truss will be about 0.4 tons per running foot.

Fig. 5.

GROUND PLAN.



The angle α is about 57°, and therefore $\frac{1}{\sin \alpha} = 1.22$.

The greatest strain on all diagonals of one panel will take place at the ends of centre span. It will be

$$S = \frac{280}{2} \times 1.22 \times 0.4 = 68.32 \text{ tons.}$$

We have six diagonals in one panel, of which we consider three only to be efficient at one time,* and, if we allow 5 tons per square inch of wrought iron, the greatest area required for one diagonal will be

$$\frac{68.32}{3 \times 5} = 4.6 \text{ square inches.}$$

At the ends of the side spans of 120 feet, the area required for one diagonal will be

$$\frac{120}{2} \times 1.22 \times 0.4 \times \frac{1}{3 \times 5} = 2.0 \text{ square inches.}$$

The strain on diagonals decreases equally towards the centres of spans, where it becomes = 0.

All this is calculated under the supposition that the wind comes with its maximum force normal to the bridge, and on all parts equally. If

* The braces, consisting of small round iron, are not able to resist any considerable amount of compression, so that the three braces bearing (by their direction) *tensile* strain, will have to resist the whole force.

the wind comes with its maximum force only on one part of the superstructure, the strains on diagonals generally will be smaller than above calculated, but they will not decrease equally from both sides of spans, and will therefore not be $= 0$ in the centres. The areas towards the centres are therefore made a little larger than required for an equally distributed wind.

STRAINS IN THE LONGITUDINAL MEMBERS.

These members in the first truss against wind (that formed by bracing between the ribs) are the arches. For 100 feet from each side of span all four ribs are braced together; on the centre piece of 318 feet only the inner arches are acting. We shall now consider the centre piece as a truss supported freely on both ends, and then calculate the end pieces like beams fixed at one end, and carrying an equally distributed load, and a load (transmitted from centre piece) on the other end.

CENTRE PIECE.

The distance between the inner ribs (supposed concentrated in their centres) is 13 8 feet. The greatest strain, T, in one rib will take place in the centre, and is determined by the equation

$$T = \frac{1}{13.8} \times \frac{318^2}{8} \times q; \text{ or, with } q = 1 \text{ ton,}$$

$$T = 916 \text{ tons.}$$

END PIECES.

The distance between centres of outer ribs and inner ribs is 15.1 feet, which gives a distance between outer ribs of $2 \times 15.1 + 13.8 = 44.0$ feet.

If T is the strain on one inner rib, and T_1 on one outer rib, we have

$$T = \frac{6.9}{22} T_1.$$

For the greatest strains, T_1' and T', at the piers, we nave the equation

$$T_1' . 44 + T' . 13.8 = q . \frac{318}{2} . 100 + q \frac{100^2}{2},$$

$$T_1' \left(44 + 13.8 . \frac{6.9}{22}\right) = q\,[15900 + 5000]; \text{ and, with } q = 1 \text{ ton,}$$

$$T_1' = \frac{20900}{48\,33} = 432 \text{ tons.}$$

During such a heavy tornado the bridge will not be loaded. The strain on one rib (2 members) by the weight of superstructure is about

700 tons pressure. By a tornado, this pressure can be increased to $700 + 916 = 1616$ tons per rib, or 808 tons per member; or, diminished to $700 - 916 = -216$ tons, which gives for one member a tension of 108 tons. As the influence of temperature can cause an additional strain of about 250 tons tension or compression, we get as maximum strains in one member during a tornado

1058 tons compression, and 358 tons tension.

Considering that the suppositions we have made above are extreme, and that such a tornado will hardly occur once in twenty years, and also that the outside ribs and the truss formed by the timbers of upper roadway help somewhat to resist the wind, we will allow 15 tons per square inch for compression, and 12.5 tons for tension; and we thus obtain for the required area of one member on account of wind,

against compression, 70.53 square inches,
against tension, 28.64 square inches.

As the sectional area of one member, adopted according to the calculation for load and temperature for the centre of bridge, is 72 square inches, we do not need to take more on account of wind; but in designing the joints for the middle ribs we have to consider the influence of wind, which requires in centre of span 28.64 square inches efficient area against tension, while at the same place, by load and temperature, no tensile strain can be created.

The members of the truss under the railroad tracks are formed by longitudinal timbers (AB, CD, EF and GH, in Fig. 5, p. 47). If we call for any panel

T the strain on inner members, CD or EF,
T_1 the strain on outside members, AB or GH, we have

$$T = \frac{6.9}{13.8 + 6.9} T_1 = \tfrac{1}{3} T_1;$$

and for the centre of middle span, where T and T_1 attain their greatest values,

$$T_1 . 41.4 + T . 13.8 = \frac{280^2}{8} . 0.4,$$

$$T_1 \left(41.4 + \frac{13.8}{3}\right) = 3920,$$

$$T_1 = \frac{3920}{46.0} = 85.2 \text{ tons},$$

$$T = \frac{T_1}{3} = 28.4 \text{ tons}.$$

Allowing 1000 pounds, or 0.5 ton compression per square inch (about $\frac{1}{8}$ of crushing strain of the timber), we get for the areas required in centre of span,

for outside members, 170 square inches, or 12″ by 14″,
for inner members, 57 square inches, or 12″ by 5″.

For the greatest strains in side spans, we have the equations,

$$T_1 = \frac{1}{46\,0} \times \frac{120^2}{8} \times 0.4 = 15.7 \text{ tons},$$

$$T = \frac{T_1}{3} = 5.2 \text{ tons},$$

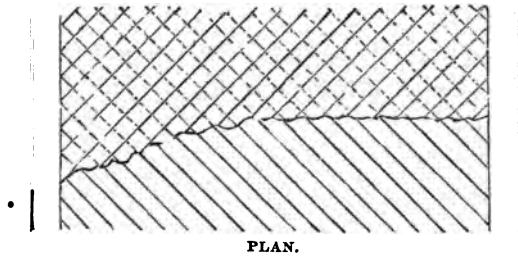
and the required areas,

for outside members, 31.4 square inches, or 12″ by 2.6″,
for inside members, 10.4 square inches, or 12″ by 1″.

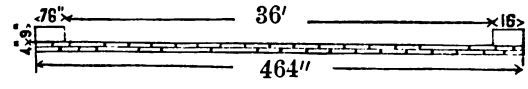
To prevent trembling from a passing train, these members have been drawn 12″ by 5″ throughout the whole bridge, and the foregoing calculation shows that, on account of wind, the outside members only, near the centre of span, must be enlarged to 12″ by 14″.

The *truss under upper roadway* is formed by two layers of planks, each 2″ thick, and lying under an angle of 45° with the direction of the bridge, one across the other (as indicated by Fig. 6). The two layers are fixed one to the other by spikes, and are on both sides screwed to longitudinal timbers.

Fig. 6.



PLAN.



SECTION.

For the calculation, we shall consider the truss as a beam of the dimensions shown in the section above. This beam continues through

the whole span, but in the centre it is supported by the arches, and we therefore assume its free span as $\frac{520}{2} = 260$ feet $= 3120$ inches.

Suppose the force of wind taken up by this beam to be 0.5 ton per running foot, or $\frac{1000}{12}$ pounds per running inch, the moment of inertia of the section is, in inches,

$$I = \frac{13 \cdot 464^3 - 9 \cdot 432^3}{12} = 47755947.$$

For the greatest tension or compression ($= K$) per square inch, we have the equation,

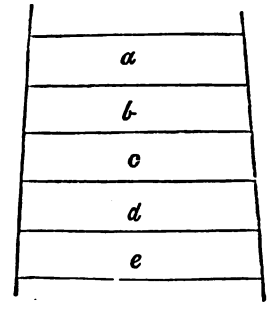
$$K = \frac{1000}{12} \cdot \frac{3120^2}{8} \cdot \frac{232}{47755947} = 492.6 \text{ pounds},$$

or about $\frac{1}{12}$ of the crushing strain of the timber.

ABUTMENTS AND PIERS.

We consider the abutments and piers (masonry laid in horizontal courses) as consisting of single plates, a, b, c, d, e, . . . with horizontal

Fig. 7.



planes at their top and base, lying one above the other, and held together only by the friction resulting from the pressure between two plates.

The abutment or pier can give way,

1st. By a sliding of the part above a horizontal joint over the part below. This will take place, if the horizontal component of the resultant of all forces acting on the piece above the joint is greater than the vertical component of the same resultant multiplied by the coefficient of friction.

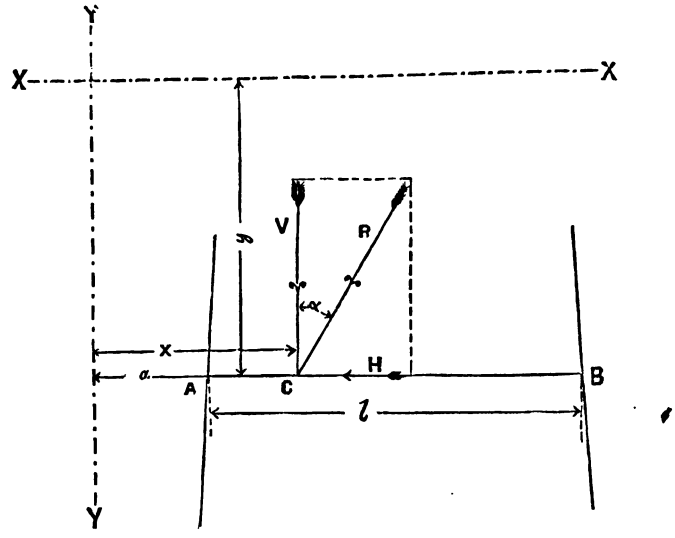
2nd. By an overthrowing of the masonry above a horizontal joint. This will take place if the resultant of all forces acting on the piece above the joint meets the plane of the joint outside the abutment or pier.

3d. By a crushing of the stone by an excessive pressure per unit of sectional area.

We shall assume that each rib is supported by a strip of masonry of the profile of the abutment or pier, and 12 feet wide. The part above the spring courses has only to carry its own weight, and we need not

examine its stability. But on the part below the spring-line we have acting, not only the weight of masonry, but also the forces exerted by the superstructure, and the pressure of earth and water surrounding the abutment or pier. We shall neglect the pressure of earth; for since this pressure would act against sliding and overthrowing, neglecting it will not diminish the safety of the following calculation. If we do not suppose that the water can enter into the joints, its pressure will be chiefly horizontal, and, acting from both sides, will have no tendency to change the value and position of the resultant R. We shall, therefore, neglect it in the following calculation. But it is possible that some water may come under the foundation, and act from below with a hydrostatic pressure, this case will be considered hereafter.

Fig. 8.



For a horizontal joint AB (Fig. 8), of the width l, at the depth y below spring-line XX, let

R = the resultant of all forces acting on the masonry above that joint,

V = the vertical component of R,

H = the horizontal component of R,

x = the distance of the point C (where R meets the plane AB) from a vertical plane, YY, perpendicular to the direction of the bridge,

α = the angle of R with a vertical line.

The coefficient of friction for masonry is about 0.7.

Then, no *sliding* will take place in the joint AB, if

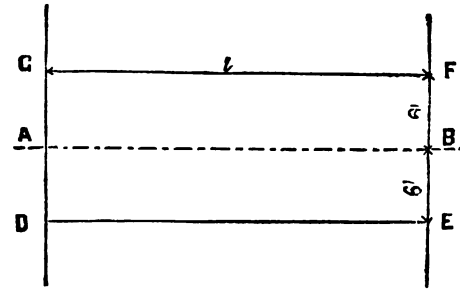
$$0.7 \times V > H.$$

No *overturning* will take place, if

$$x > a, \text{ and } x < a + l.$$

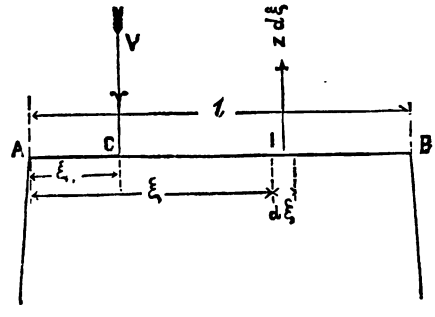
The pressures per sectional unit resulting from R can be obtained nearly enough by the following considerations.

Fig. 9.



Let *DEFG* (Fig. 9) be a horizontal section through the piece of masonry supporting one rib, and suppose the rib to come against its centre. As the resultant of the weight of the single courses lies also in the vertical plane through the centre, we can assume all forces as acting in that plane. All pressures, in the section *DEFG* will therefore be symmetrically arranged with respect to the centre line AB.

Fig. 10.



Let Fig. 10 be a vertical section through AB (as on p. 53),

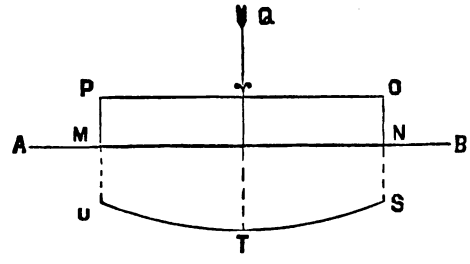
l = the width of a course AB at the depth y below spring-line,
ξ = the distance of any point I of AB from A,
ξ_1 = the distance of the point C from A [$\xi_1 = x - a$],
$z d\xi$ = the resultant of all vertical pressures acting on a strip of the infinitely small length $d\xi$, and extending over the whole width of 12 feet.

Then we have the equations

$$(\mathrm{I}) \quad \int_0^l z d\xi = V,$$

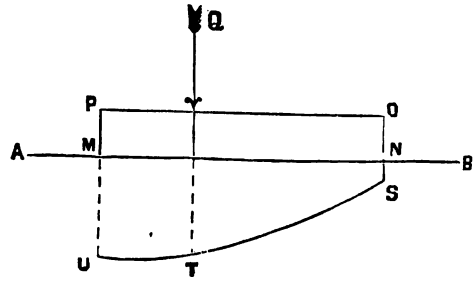
$$(\mathrm{II}) \quad \int_0^l z \cdot \xi d\xi = V\xi_1.$$

Fig. 11.



If an elastic plate (*MNOP*) be pressed against a plane (*AB*), by a force (*Q*) perpendicular to *AB*, and passing through the centre of *MNOP*, it is plain that the greatest pressure on *AB* would be exerted in the centre of *MN*, and that the forces acting along *MN*, if represented by ordinates, would form a certain curve, *STU*, symmetrical with respect to *Q*. But the thicker the plate *MNOP*, and the more rigid the material, the flatter would be that curve, and for a perfectly rigid body it would be a straight line, parallel to *AB*. But if *Q* should not

Fig. 12.



act in the centre of *MN*, the pressures represented by ordinates would

give another curve, *STU*, no longer symmetrical with respect to *Q*; and for a perfectly rigid material, *STU* would be an inclined straight line. As stone possesses great rigidity, we may assume that the distribution of the pressures, if laid down in a diagram, will give a straight line; or that z is a function of ξ of the first degree, such as

$$z = m\xi + c.$$

As all forces on the section *DEFG* (Fig. 9) act in a vertical plane through its centre line *AB*, the pressures along any line, parallel to *DG* and *EF*, are constant; and, taking a foot as the unit, z represents a force equal to the pressure at *I* (Fig. 10), acting on 12 square feet.

With $z = m\xi + c$, we obtain from equation (I)

$$m \cdot \frac{l^2}{2} + c \cdot l = V,$$

and from (II),

$$m \cdot \frac{l^3}{3} + c \cdot \frac{l^2}{2} = V\xi_1.$$

From the last two equations we deduce

$$m = -\frac{6V}{l^3}(l - 2\xi_1),$$

$$c = \frac{2V}{l^2}(2l - 3\xi_1),$$

and hence,

$$\text{(III)} \quad z = \frac{2V}{l^2}(2l - 3\xi_1) - \frac{6V}{l^3}(l - 2\xi_1)\xi.$$

We see that the pressure $\left\{\begin{matrix}\text{increases}\\ \text{decreases}\end{matrix}\right\}$ for an increasing ξ when $\xi_1 \left\{\begin{matrix}>\\ <\end{matrix}\right\} \frac{l}{2}$.

As we assume that no tensile strain can take place, this equation can only be used, if between $\xi = 0$ and $\xi = l$, it gives no negative value for z. Because z is a function of ξ of the first degree, we have only to examine the values z_0 for $\xi = 0$ and z_l for $\xi = l$. One of them is the greatest and the other the least value.

From (III) we have, for $\xi = 0$,

$$\text{(III a)} \quad z_0 = \frac{2V}{l^2}(2l - 3\xi_1),$$

which will be negative when $\xi_1 > \frac{2}{3}l$.

From (III) we have, for $\xi = l$,

$$\text{(III b)} \quad z_l = \frac{2V}{l^2}(3\xi_1 - l),$$

which will be negative when $\xi_1 < \frac{1}{3}l$.

The equations (III), (III a), and (III b,) can therefore be used only when

$$\tfrac{1}{3}l < \xi_1 < \tfrac{2}{3}l;$$

for $\xi_1 < \frac{1}{3}l$ and $\xi_1 > \frac{2}{3}l$ only a part of the bed of the course is efficient. The pressures z can be represented in these two cases by the ordinates of the line BC in Fig. 13 and DF in Fig. 14.

Fig. 13. ($\xi_1 < \frac{1}{3}l$).

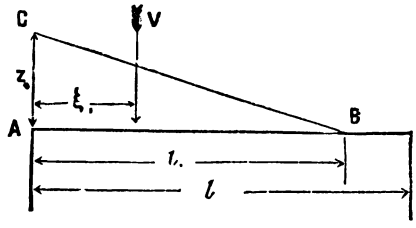
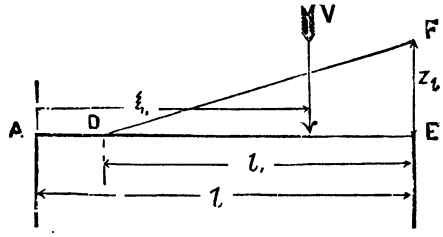


Fig. 14. ($\xi_1 > \frac{2}{3}l$).



The portion of the course which is sustaining the pressure will have the length l_1, which we can find in the following manner:

The conditions of equilibrium give us the equations

$$(4)\quad \int_0^{l_1} z\,d\xi = V \text{ (for } \xi_1 < \tfrac{1}{3}l), \qquad (IV)\quad \int_{l-l_1}^{l} z\,d\xi = V \text{ (for } \xi_1 > \tfrac{2}{3}l),$$

and

$$(5)\quad \int_0^{l_1} z\,\xi\,d\xi = V\xi_1 \text{ (for } \xi_1 < \tfrac{1}{3}l), \qquad (V)\quad \int_{l-l_1}^{l} z\,\xi\,d\xi = V\xi_1 \text{ (for } \xi_1 > \tfrac{2}{3}l).$$

The integrals in equations (4) and (IV) are the areas of the triangles ABC, Fig. 13, and DEF, Fig. 14, and therefore

$$(4a)\quad z_0 \frac{l_1}{2} = V \text{ (for } \xi_1 < \tfrac{1}{3}l), \qquad (IV\,a)\quad z_l \cdot \frac{l_1}{2} = V \text{ (for } \xi_1 > \tfrac{2}{3}l).$$

The integrals of (5) and (V) are the static moments of the triangles ABC and DEF around the point A. As the centres of gravity lie at one third of l_1 from AB and EF, we have

(5a) $z_0 \cdot \frac{l_1}{2} \cdot \frac{l_1}{3} = V\xi_1$ (for $\xi_1 < \frac{1}{3}l$), (Va) $z_l \frac{l_1}{2}\left(l - \frac{l_1}{3}\right) = V\xi_1$ (for $\xi_1 > \frac{2}{3}l$).

From (4a) and (5a), and (IVa) and (Va), we get

(6) $l_1 = 3\xi_1$ (for $\xi_1 < \frac{1}{3}l$), (VI) $l_1 = 3(l - \xi_1)$ (for $\xi_1 > \frac{2}{3}l$),

and (7) $z_0 = \frac{2V}{3\xi_1}$ (for $\xi_1 < \frac{1}{3}l$), (VII) $z_l = \frac{2V}{3(l-\xi_1)}$ (for $\xi_1 > \frac{2}{3}l$).

If V is given in tons and l and ξ_1 in feet, z is the vertical pressure per 12 square feet. To obtain the vertical pressures p in ponnds per square inch, we have

$$p = \frac{2000}{1728} \cdot z,$$

and the pressures in the direction of R (which are the greatest) will be

$$\frac{p}{\cos \alpha}.$$

The values of V, H, x, ξ_1 and α depend upon the figure of the abutment or pier, upon the weight of the material used in it, and upon the forces exerted by the superstructure.

A. Abutments.

Fig. 15 gives the form and dimensions of the west abutment.* We consider a strip of masonry 12′ wide, supporting one rib.

The forces exerted on the abutment by this rib can be regarded as consisting of

a vertical force P through the centre of the base of arch,
a horizontal force Q through the same point, and
a moment Na (or couple of forces).

Let us now put

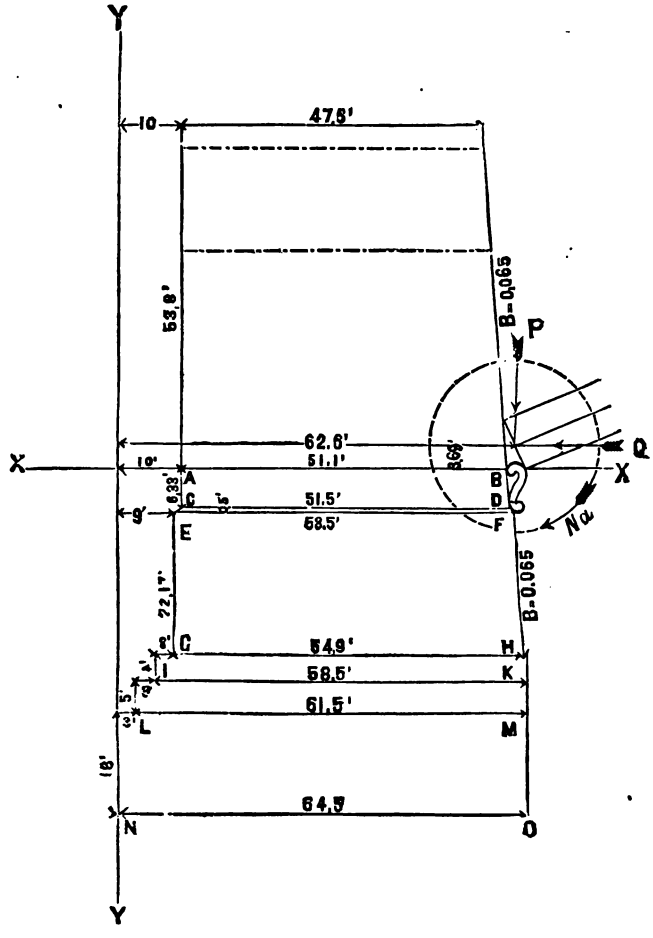
W = the weight of masonry above the spring-line XX, in tons,
a' = distance of the centre of gravity of the masonry above spring-line, from YY, in feet,
q = weight of 12 cubic feet masonry (1 square foot of section), in tons.

The horizontal force is constant for the whole pier below the spring-line, and we have throughout

$$H = Q.$$

* The east abutment, resting on a pile foundation, has a much larger base than the west abutment, which rests immediately on the rock, and no separate calculation as to its stability is therefore required.

Fig. 15.



The vertical component V is

for the spring line AB, $y=0$,

$$V_0 = W + P;$$

between AB and CD,

$$V = V_0 + q\,y\,(51.1 + \tfrac{1}{2} \times 0.065\,y);$$

for CD, $y=6.33'$,

$$V_1 = V_0 + q\,(6.33)\,(51.1 + \tfrac{1}{2} \times 0.065 \times 6.33);$$

from CD to EF,

$$V = V_1 + q\,(y - 6.33)\,[51.5 + 2\,(y - 6.33)];$$

for EF, $y=6.83'$,

$$V_2=V_1+q\,(0.5)\,(51.5+2\times0.5);$$

from EF to GH,

$$V=V_2+q\,(y-6.83)\,[53.5+\tfrac{1}{2}\times0.065\,(y-6.83)];$$

for GH, $y=29.0'$,

$$V_3=V_2+q\,(22.17)\,[53.5+\tfrac{1}{2}\times0.065\times22.17];$$

from GH to IK,

$$V=V_3+q\,(y-29.0)\times58.5;$$

for IK, $y=33'$,

$$V_4=V_3+q\times4\times58.5;$$

from IK to LM,

$$V=V_4+q\,(y-33)\times61.5;$$

for LM, $y=38'$,

$$V_5=V_4+q\times5\times61.5;$$

from LM to NO,

$$V=V_5+q\,(y-38)\times64.5;$$

for NO, $y=54'$,

$$V_6=V_5+q\times16\times64.5.$$

For the calculation of x we have the equations:

For spring-line AB, $y=0$,

$$x_0=\frac{1}{V_0}\,[Wa'+P.62.6+Na-Q\times3.69];$$

from AB to CD,

$$x=\frac{1}{V}\left[V_0x_0-Qy+q\int_0^y(51.1+0.065y)\left(10+\frac{51.1\times0.065y}{2}\right)dy\right];$$

or because the batter (=0.065) is but small, nearly enough

from AB to CD,

$$x=\frac{1}{V}\left[V_0x_0-Qy+q\,.\,y\,(51.1+\tfrac{1}{2}\times0.065y)\left(10+\frac{51.1+\tfrac{1}{2}\times0.065\,y}{2}\right)\right];$$

for CD, $y=6.33'$,

$$x_1=\frac{1}{V_1}\left[V_0x_0-Q\,(6.33)+q\,(6.33)\,(51.1+\tfrac{1}{2}\times0.065\times6.33)\left(10+\frac{51.1+\tfrac{1}{2}\times0.065\times6.33}{2}\right)\right];$$

from CD to EF,

$$x=\frac{1}{V}\left[V_1x_1-Q(y-6.33)+q(y-6.33)[51.5\times2(y-6.33)]\left(10+\frac{51.5}{2}\right)\right];$$

for EF, $y=6.83'$

$$x_2=\frac{1}{V_2}\left[V_1x_1-Q(0.5)+q(0.5)\,(51.5+2\times0.5)\left(10+\frac{51.5}{2}\right)\right];$$

from EF to GH,

$$x=\frac{1}{V}\Big[V_2x_2-Q(y-6.83)$$
$$+q(y-6.83)(53.5+\tfrac{1}{2}\times 0.065[y-6.83])\Big(9+\frac{53.5+\frac{1}{2}\times 0.065(y-6.83)}{2}\Big)\Big];$$

for GH, $y=29.0'$,

$$x_3=\frac{1}{V_3}\Big[V_2x_2-Q(22.17)$$
$$+q(22.17)\,(53.5+\tfrac{1}{2}\times 0.065\times 22.17)\Big(9+\frac{53.5+\frac{1}{2}\times 0.065\times 22.17}{2}\Big)\Big];$$

from GH to IK,

$$x=\frac{1}{V}\Big[V_3x_3-Q(y-29.0)+q(y-29.0)\,(58.5)\Big(6+\frac{58.5}{2}\Big)\Big];$$

for IK, $y=33'$,

$$x_4=\frac{1}{V_4}\Big[V_3x_3-Q(4)+q(4\times 58.5)\Big(6+\frac{58.5}{2}\Big)\Big];$$

from IK to LM,

$$x=\frac{1}{V}\Big[V_4x_4-Q(y-33)+q.(y-33)\,(61.5)\Big(3+\frac{61.5}{2}\Big)\Big];$$

for LM, $y=38'$,

$$x_5=\frac{1}{V_5}\Big[V_4x_4-Q(5)+q(5\times 61.5)\Big(3+\frac{61.5}{2}\Big)\Big];$$

from LM to NO,

$$x=\frac{1}{V}\Big[V_5x_5-Q(y-38)+q(y-38)\times 64.5\times\frac{64.5}{2}\Big];$$

for NO, $y=54'$,

$$x_6=\frac{1}{V_6}\Big[V_5x_5-Q(16)+q(16)\times 64.5\times\frac{64.5}{2}\Big].$$

For ξ_1 we have the equations:

from AB to CD	$\xi_1=x-10$;
from CD to EF,	$\xi_1=x-10+2(y-6.33)$;
from EF to GH (inclusive),	$\xi_1=x-9$;
below GH to IK (inclusive),	$\xi_1=x-6$;
below IK to LM (inclusive),	$\xi_1=x-3$;
from LM to NO,	$\xi_1=x$.

The angle α is determined by the equation

$$\tan\alpha=\frac{H}{V}.$$

The quantity of masonry above the spring-line, for all four ribs, if solid, would be in round numbers

$$= \frac{51 + 47.6}{2} \times 53.8 \times 48 = 127312 \text{ cub. feet.}$$

The cubic contents of the two openings for the passage of the railroads are

$$= 19200 \text{ cub. feet.}$$

Subtracting this from the above number, there remains 108112 cubic feet; and we now have for one rib (strip of masonry 12′ wide)

$$\frac{108112}{4} = 27028 \text{ cub. feet.}$$

Neglecting the fact that the greater portion of the spring-courses is composed of granite (180 lbs. per cub. ft.) and assuming the weight of one cub. ft. of the masonry at 150 lbs. =0.075 tons, we have the weight of masonry above spring-line, for one rib,

$$W = 27028 \times 0.075 = 2027.1 \text{ tons.}$$

The distance of the centre of gravity of that weight from YY is, approximately,

$$a' = 10 + \frac{47.6 + 51.1}{4} = 34.7 \text{ feet.}$$

If we assume for the masonry below the spring-line also a weight of 150 lbs. = 0.075 tons per cubic foot, we get for q,

$$q = 12 \times 0.075 = 0.9 \text{ ton.}$$

The effect of the superstructure is the more dangerous for the abutment, the greater Q, and the smaller P and Na.

The greatest value of Q occurs when P also has its greatest value, and P will be the smallest when Q has its smallest value. The influence of P is only small in comparison with that of Q, and therefore it will be sufficient to make a computation for the two cases in which

1. Q has its greatest, and
2. Na has its smallest (greatest negative) value.

1. Q has its greatest value for a movable load over the whole span, combined with the highest temperature.

The forces exerted on the abutment by the arch are for this case (if we suppose all arches having the dimensions of those of central span),

	Horizontal Forces.	Vertical Forces.	Moments.
From weight of bridge,	∴652.0 tons	∴259.0 tons	∴— 340 foot-tons
" movable load,	∴521.6 "	∴207.2 "	∴— 272 "
" temperature,	∴103.6 "	∴ 0.	∴—3736 "
	Q=1277.2 "	P=466.2 "	Na=—4348 "

In the following table, the values of V, x, ξ_1, tan α are obtained from the formulæ pp. 59, 60 & 61.

l is the width of the abutment in the depth y below spring-line, and is determined by the equations:

from AB to CD,	$l = 51.1 + 0.065\,y$;
from CD to EF,	$l = 51.5 + 4\,(y - 6.33)$;
from EF to GH,	$l = 53.5 + 0.065\,(y - 6.83)$;
below GH to IK,	$l = 58.5$;
below IK to LM,	$l = 61.5$;
below LM to NO,	$l = 64.5$.

l_1, for $\xi_1 < \frac{1}{3}l$ and $\xi_1 > \frac{2}{3}l$, is the efficient length of joint, and is determined by equations (6) and (VI) (p. 58).

p_0 and p_l are the vertical pressures per square inch on the left and right end of the joint, and are obtained from the equations

$$p_0 = \frac{2000}{1728} z_0, \qquad p_l = \frac{2000}{1728} z_l,$$

z_0 and z_l being determined by equations (III a) and (III b) (p. 56), respectively by equations (7) and (VII) (p. 58).

$\frac{p_0}{\cos \alpha}$ and $\frac{p_l}{\cos \alpha}$ are the pressures per square inch in the direction of the resultant R, at the ends of the joint.

TABLE I.

y in feet	V in tons	d' in feet	ξ_1 in feet	l in feet	l_1 in feet	p_0 pounds per □″	p_l pounds per □″	tan α	$\frac{p_0}{\cos \alpha}$ pounds per □″	$\frac{p_l}{\cos \alpha}$ pounds per □″
0 (*AB*)	2493.3	36.29	26.29	51.1	.....	51.6	61.4	0.512	58.0	69.0
6.33 (*CD*)	2785.6	33.32	23.32	51.5	.....	80.3	44.9	0.458	88.3	49.4
6.83 (*EF*)	2809.2	33.11	24.11	53.5	.....	78.8	42.8	0.455	86.6	47.0
14.0	3155.9	30.51	21.51	53.97	.....	108.8	26.5	0.405	117.4	28.6
21.0	3497.4	28.50	19.50	54.42	.....	137.5	11.2	0.365	146.4	11.9
29.0 (*GH*)	3891.1	26.66	17.66	54.90	52.98	170.0	.....	0.328	178.9	.....
33.0 (*IK*)	4101.7	25.85	19.85	58.5	.....	159.4	2.9	0.311	166.9	3.0
38.0 (*LM*)	4378.4	24.89	21.89	61.5	.....	153.6	11.2	0·292	160.0	11.7
46.0	4842.8	23.49	23.49	64.5	.....	157.7	16.1	0.264	163.1	16.7
54.0 (*NO*)	5307.3	22.34	22.34	64.5	.....	183.0	7.4	0.241	188.2	7.6

2. The least (greatest negative) value of Na occurs at the highest temperature, if $\frac{3}{8}$ of span measured from the abutment are unloaded, and the other $\frac{5}{8}$ are loaded. Under these circumstances, we have

	Horizontal Forces.	Vertical Forces.	Moments.
From weight of bridge,	$\therefore$ 652.0 tons	$\therefore$ 259.0 tons	$\therefore$ — 340 foot-tons
" movable load,	$\therefore$ 380.4 "	$\therefore$ 68.5 "	$\therefore$ —4266 "
" temperature,	$\therefore$ 103.6 "	$\therefore$ 0.	$\therefore$ —3736 "
	Q=1136.0 "	P=327.5 "	Na=—8342 "

For these values, Table II is calculated in the same manner as Table I.

TABLE II.

y in feet	V in tons	x in feet	ξ_1 in feet	l in feet	l_1 in feet	p_0 pounds per □″	p_1 pounds per □″	tan α	$\frac{p_0}{\cos\alpha}$ pounds per □″	$\frac{p_1}{\cos\alpha}$ pounds per □″
0 (*AB*)	2354.6	33.26	23.26	51.1	.....	67.6	39.0	0.482	75.1	43.3
6.33 (*CD*)	2646.9	30.81	20.81	51.5	.....	93.8	25.2	0.429	102.0	27.4
6.83 (*EF*)	2670.5	30.64	21.64	53.5	.....	91.0	24.6	0.426	98.8	26.8
14.0	3017.2	28.54	19.54	53.97	.....	118.2	11.0	0.377	126.3	11.8
21.0	3358.7	26.94	17.94	54.42	53.82	144.6	.....	0.338	152 5	.....
29.0 (*GH*)	3752.4	25.51	16.51	54.9	58.53	173.9	.....	0.303	181.7	.....
33.0 (*IK*)	3963.0	24.87	18.87	58.5	56.61	162.1	.....	0.287	168.7	.....
38.0 (*LM*)	4239.7	24.10	21.11	61.5	.....	154.8	4.7	0.268	160.3	4.9
46.0	4704.1	22.98	22.98	64.5	.....	157.3	11.6	0.242	161.9	11.9
54.0 (*NO*)	5168.6	22.06	22.06	64.5	.....	180.7	4.8	0.220	184.9	4.9

We see from Tables I and II that there is no danger

1. Of *sliding*, because tan $\alpha = \frac{V}{H}$ is always < 0.7;

2. Of *overturning*, because always $\xi_1 > 0$, and $\xi_1 < l$;

3. Of *crushing*, because the greatest normal pressure is only 192.2 lbs. per square inch, while the crushing strain of good limestone is about 6000 lbs. per square inch.

For the case when the water comes under the foundation, and acts from below with the hydrostatic pressure (which we will assume as acting on the whole area of the bottom NO) we have two kinds of pressures, which together resist the resultant R_6.

One of them, the hydrostatic pressure, is equally distributed and vertical; and its resultant is therefore a vertical force acting upwards in the centre of the joint NO. If the height of water above NO be h, the hydrostatic pressure per linear foot of joint will be $\zeta = \frac{h\,6.25}{2000} \times 12$, and the resultant of it on the whole joint will be $64.5 . \zeta$.

The other kind of pressures is exerted on the foundation course by the solid materials of the bottom. If we call the resultant of those pressures R', and understand by V', H', x', ξ', etc, the same quantities for R', which V, H, x, ξ, etc., are for R, we have the equations

$$V' = V_6 - 64.5\,l, \qquad H' = H = Q,$$
$$\xi_1' = x' = \frac{1}{V'}\left[V_6 x_6 - 64.5\,\zeta . \frac{64.5}{2}\right].$$

For the determination of z_0', z_l', l_1', p_0', p_l', $\tan \alpha'$, the equations given for z_0, z_l, l_1, &c., can be used by putting V' and ξ_1', instead of V and ξ_1.

For the highest stage of water, which we assume to be at the spring-line, we have $h = 54$ feet, and

$$\zeta = \frac{54 \times 62.5}{2000} \times 12 = 20.25 \text{ tons.}$$

The greatest V_6 we find from Table I to be 5307.3 tons, and with that

$$V' = 5307.3 - 64.5 \times 20.25 = 5307.3 - 1306.1 = 4001.2 \text{ tons.}$$

$$\xi_1' = x' = \frac{1}{4001.2}\left[5307.3 \times 21.90 - 1306.1 \times \frac{64.5}{2}\right] = \frac{1}{4001.2} \times 74108.2$$
$$= 18.52'.$$

Because $\xi_1' < \frac{l}{3}$, we have

$$l_1' = 3\xi_1' = 55.56',$$
$$p_1' = \frac{2000}{1728} \cdot z_0' = \frac{2000}{1728} \times \frac{2 \times 4001.2}{3 \times 18.52},$$
$$= 166.7 \text{ pounds per square inch,}$$
$$\tan \alpha' = \frac{1277.2}{4001.2} = 0.319,$$
$$\text{and } \frac{p_0'}{\cos \alpha} = 175.0 \text{ pounds per square inch.}$$

The normal pressure of water is at that depth $= \frac{2000}{1728} . \zeta = 23.4$ lbs. per square inch.

For the greatest total pressure at the left end of the joint we now get $175.0 + 23.4 = 198.4$ lbs. per square inch, or about $\frac{1}{30}$ of the crushing strain of the stone.

B. Piers.

The east pier is the higher of the two, and it will be sufficient to establish its stability, since all dimensions, except the depth, are the same in both piers, and the same external forces act on them. We suppose for greater simplicity that the two arches are of the same dimensions.

We have for the whole part below spring-line XX

$$H = Q_1 - Q.$$

Further:

for AB, $y = 0$, $V_0 = W + P + P_1$;
from AB to CD, $V = V_0 + q \cdot y\,[31.66 + 0.065\,y]$
for CD, $y = 6.33$, $V_1 = V_0 + q\,(6.33)\,\frac{32.5 + 31.66}{2}$;
from CD to EF, $V = V_1 + q\,(y - 6.33)\,[32.5 + 2\,(y - 6.33)]$;
for EF, $y = 6.83$, $V_2 = V_1 + q\,(0.5)\,[32.5 + 2 \times 0.5]$;
from EF to GH, $V = V_2 + q\,(y - 6.83)\,[34.5 + 0.065\,(y - 6.83)]$;
for GH, $y = 106.83$, $V_3 = V_2 + q\,(100)\,[34.5 + 0.065 \times 100]$;
from GH to IK, $V = V_3 + q(y - 106.83)[56.0 + 0.065 \cdot (y - 106.83)]$;
for IK, $y = 132$, $V_4 = V_3 + q\,(25.17) \cdot [56.0 + 0.065 \times 25.17]$.

For x, we have, in general,

$$x = \frac{1}{V}[V_0 x_0 - (Q_1 - Q)\,y];$$

V being found for any depth by the preceding formulæ.

For l we have the equations:

from AB to CD, $l = 31.66 + 0.13\,y$;
from CD to EF, $l = 32.50 + 4(y - 6.33)$;
from EF to GH, $l = 34.50 + 0.13\,(y - 6.83)$;
from GH to IK, $l = 56.0 + 0.13\,(y - 106.83)$.

For ξ_1 we have the equation

$$\xi_1 = \frac{l}{2} + x.$$

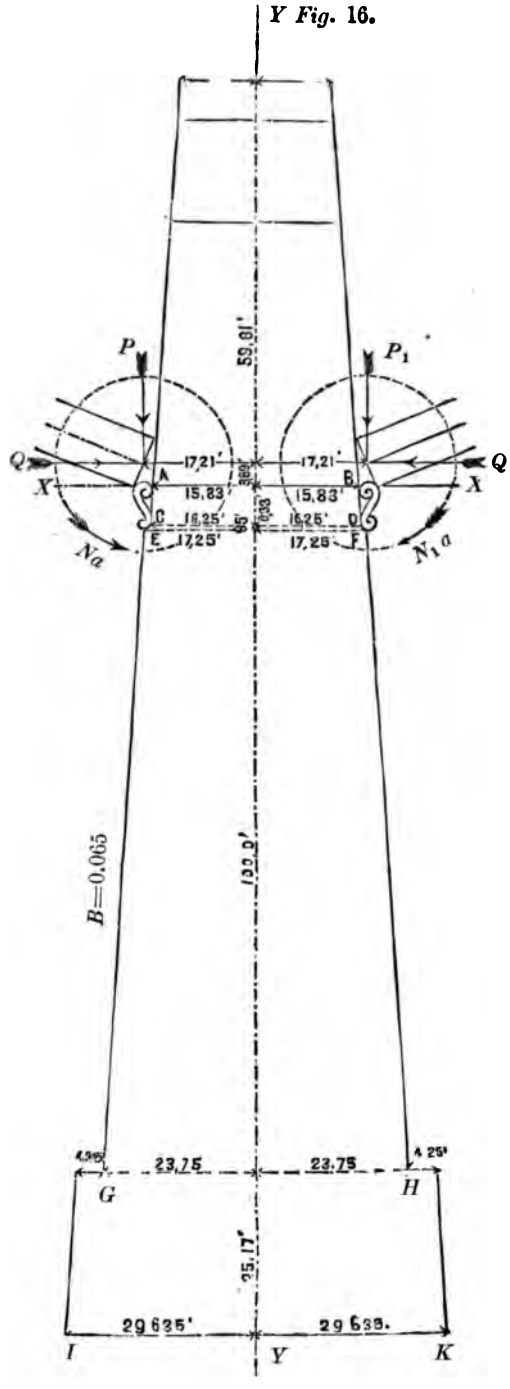
The quantity of masonry above spring-line, for a strip 12′ wide, is

$$= 18478 \text{ cub. feet},$$

and the weight of it, $W = 18478 \times 0.075 = 1386$ tons.

As for the abutment, we take $q = 0.9$ ton.

The forces $(Q_1 - Q)$, $(P - P_1)$ and $(Na - N_1 a)$ have (if positive) a tendency to overturn the pier from the right to the left. The force $(P_1 + P)$ increases the vertical pressure.


Y Fig. 16.
P
P1
Q
Q1
X
X
59.81'
17,21'
17,21'
15,83'
15,83'
A
B
C
D
E
F
16,25'
16,25'
17,25'
17,25'
3.80'
0.33
95'
Na
N1a
B=0,065
132.9'
23,75
23,75
4,25'
4,25'
G
H
25,17'
29,625'
29,625.
I
Y
K

Since, for overturning, the influence of $(P - P_1)$ is small in comparison with that of $(Q_1 - Q)$, it will be sufficient to make calculations for the following cases:

1. For the greatest value of $(Q_1 - Q)$,
2. For the greatest value of $(Na - N_1 a)$,

} to establish the safety against sliding, overturning and crushing on one side;

3. For the greatest value of $(P_1 + P)$;

} to establish the safety against crushing by a too great vertical pressure.

1. We suppose that the pier has to sustain the thrust of one unloaded arch only, at a high temperature (which case can occur in building the bridge, or by an accident).

We have in this case Q, P and $Na = 0$, and

$$Q_1 = 652.0 + 103.6 = 755.6 \text{ tons},$$
$$P_1 = 259.0 \text{ tons}.$$
$$N_1 a = -340 - 3736 = -4076 \text{ tons}.$$

The following table is calculated for these values in the same manner as the tables for the abutment.

TABLE I.

y in feet	V in tons	x in feet	ξ_1 in feet	l in feet	l_1 in feet	p_0 pounds per □″	p_1 pounds per □″	tan α	$\frac{p_0}{\cos \alpha}$ pounds per □″	$\frac{p_1}{\cos \alpha}$ pounds per □″
0.00, AB	1645.0	— 1.46	14.37	31.66	.....	76.8	43.5	0.459	84.5	47.9
6.33, CD	1827.8	— 3.92	12.32	32.50	.....	112.3	17.9	0.413	121.5	19.3
6.83, EF	1842.8	— 4.10	13.15	34.50	.....	105.9	17.7	0.410	114.5	19.2
31.83	2655.6	— 9.96	8.92	37.75	26.76	229.7	.....	0.285	238.8	.....
56.83	3541.6	—12.80	7.70	41.00	23.10	354.9	.....	0.213	362.9	.....
81.83	4500.6	—14.27	7.86	44.25	23.58	441.8	.....	0.168	448.0	.....
106.83, GH	5532.8	—15.02	8.73	47.50	26.19	489.0	.....	0.136	493.6	.....
132.00, IK	6838.5	—14.93	14.70	59.27	44.10	358.9	.....	0.110	361.1	.....

2. We get the greatest value of $(Nu - N_1 a)$, if on the left side of the pier $\frac{3}{8}$ of span (measured from the pier) are loaded, and the other $\frac{5}{8}$ of span unloaded and if on the right side $\frac{3}{8}$ of span, measured from the pier, are free of load, and the other $\frac{5}{8}$ are loaded. The influence of

temperature acts equally from both sides, and has no effect on the part below the spring-line.

For this case,

$Q_1 = 652.0$ (weight of bridge) $+ 380.4$ (load) $= 1032.4$ tons,
$P_1 = 259.0$ " $+ 68.5$ " $= 327.5$ tons,
$N_1 a = -340$ " -4266 " $= -4606$ foot-tons,

$Q = 652.0$ " $+ 141.3$ " $= 793.3$ tons,
$P = 259.0$ " $+ 138.7$ " $= 397.7$ tons,
$Na = -340$ " $+ 3994$ " $= +3654$ foot-tons.

TABLE II.

y in feet	V in tons	x in feet	ξ_1 in feet	l in feet	l_1 in feet	p_0 pounds per □″	p_l pounds per □″	tan α	$\frac{p_0}{\cos \alpha}$ pounds per □″	$\frac{p_l}{\cos \alpha}$ pounds per □″
0.00 (AB)	2111.2	—4.89	10.94	31.66	.....	148.7	5.7	0.113	149.6	5.7
6.33 (CD)	2294.0	—5.17	11.08	32.50	.....	159.7	3.7	0.104	160.5	3.7
6.83 (EF)	2309.0	—5.19	12.06	34.50	.....	147.4	7.5	0.104	148.2	7.5
31.83	3121.8	—5.75	13.13	37.75	.....	183.1	8.4	0.077	183.6	8.4
56.83	4007.8	—5.97	14.53	41.00	.....	212.0	14.3	0.060	212.2	14.3
81.83	4966.9	—6.02	16.11	44.25	.....	235.9	23.9	0.048	236.2	24.0
106.83 (GH)	5999.0	—5.98	17.77	47.50	.....	256.6	35.7	0.040	256.8	35.8
132.00 (IK)	7304.7	—5.74	23.90	59.27	.....	225.4	59.8	0.033	225.5	59.8

3. The greatest value of $(P_1 + P)$ takes place if both arches are loaded over their whole length. In this case the resultant will always act in the vertical centre line of the pier, and the pressure per sectional unit will be the same over a whole horizontal joint. We have for this case,

$Q = Q_1 = 652.0 + 521.6 = 1173.6$ tons,
$P = P_1 = 259.0 + 207.2 = 466.2$ tons,
$Na = N_1 a = -340 - 272 = -612$ foot-tons.

We have through the whole pier,

$$x = 0, \qquad \xi_1 = \frac{l}{2}, \qquad \tan \alpha = 0.$$

$$p_0 = p_l = p = \frac{2000}{1728} \times \frac{V}{l}.$$

TABLE III.

y in feet	V in tons	l in feet	p pounds per □″
0.00 (AB)	2318.4	31.66	84.7
6.33 (CD)	2501.2	32.50	89.1
6.83 (EF)	2516.2	34.50	84.4
31.83	3329.1	37.75	102.1
56 83	4215.0	41.00	118.9
81.83	5174.0	44.25	135.3
106.83 (GH)	6206.2	47.50	151.2
132.00 (IK)	7511.9	59 27	146.7

The most dangerous case for the pier is that of Table I. The greatest value of tan α is 0.459, therefore perfect safety against sliding.

The resultant lies always within the body of masonry, therefore no danger of overturning.

The greatest pressure per square inch is 494 pounds, or only about $\frac{1}{12}$ of the crushing strain.

After the completion of the bridge, the case of Table I cannot occur, and the greatest value of $Q_1 - Q$ will take place if one span is loaded to its full capacity, and the other unloaded. For this case is

$Q_1 = 1173.6$ tons, $\quad Q = 652.0$ tons,
$P_1 = 466.2$ tons, $\quad P = 259.0$ tons,
$N_1 a = -612$ foot-tons, $\quad Na = -340$ foot-tons.

Table IV is calculated for these values.

TABLE IV.

y in feet	V in tons	x in feet	ξ_1 in feet	l in feet	l_1 in feet	p_0 pounds per □″	p_1 pounds per □″	tan α	$\frac{p_0}{\cos \alpha}$ pounds per □″	$\frac{p_l}{\cos \alpha}$ pounds per □″
0.00, AB	2111.2	+0.65	16.48	31.66	.....	58.5	74.9	0.247	60.2	77.2
6.33, CD	2294.0	—0.84	15.41	32.50	.....	81.5	59.6	0.227	83.6	61.2
6.83, EF	2309.0	—0.95	16.30	34.50	.....	78.1	55.8	0.226	79.9	57.3
31.83	3121.8	—4.88	14.00	37.75	.....	146.8	18.6	0.167	148.8	18.9
56.83	4007.8	—7.05	13.45	41.00	40.35	198.7	.....	0.130	200.4	.....
81.83	4966.9	—8.32	13.81	44.25	41.43	239.8	.....	0.105	241.1	.....
106.83, GH	5999.0	—9.06	14.69	47.50	44.07	272.2	.....	0.087	273.2	.....
132.0, IK	7304.7	—9.43	20.21	59.27	.....	240.8	5.7	0.071	241.4	5.7

We see that the greatest possible strain after the bridge is completed, is 273 pounds per square inch, or about $\frac{1}{21}$ of the crushing strain of the stone.

For the case in which water comes under the foundation, we have for a stage of water at spring-line, with the same notation as on p. 65,

$$h = 132 \text{ feet},$$

$$\zeta = \frac{132 \times 62.5}{2000} \times 12 = 49.50 \text{ tons}.$$

The efficient upward vertical pressure of the water, on the assumption that it acts upon the whole base of the pier, is simply the weight of a volume of water equal to that of the submerged part of the pier. The weight of this part of the pier at 150 lbs. per cubic foot is 5193.5 tons (= difference of first and last values of V of the first column of our tables); and the weight of the same volume of water is

$$5193.5 \times \frac{62.5}{150} = 2164 \text{ tons},$$

and the resultant downward pressure at the bottom course is

$$V' = 6838.5 - 2164 = 4674.5 \text{ tons},$$

$$x' = \frac{1}{4674.63} \times 6838.5 \times (-14.94) = -21.86 \text{ ft.},$$

$$\xi_1' = 29.64 - 21.86 = 7.78 \text{ ft.},$$

$$p_0' = \frac{2000}{1728} \times \frac{2 \times 4674.63}{3 \times 7.78} = 463.6 \text{ pounds per square inch},$$

$$\tan \alpha' = \frac{755.6}{4674.63} = 0.162,$$

$$\frac{p_0'}{\cos \alpha'} = 469.6 \text{ pounds per square inch}.$$

The greatest normal pressure at the left end of the foundation course will be

$$469.6 + \tfrac{2000}{1728} \cdot \zeta = 469.6 + 57.3 = 526.9 \text{ pounds per square inch},$$

or about $\frac{1}{11}$ of the crushing strain of stone.

Effect of the Current and of Wind on the Piers.

For the horizontal impact of an unlimited stream, Weisbach gives the formula,

$$P\zeta = \frac{v^2}{2g} F\gamma,$$

in which

P = the horizontal force (impact) exerted by the water on a body obstructing its current,
F = the area opposed to the current (= greatest section of the obstructing body, taken at right angles to the current),
v = the velocity of the stream,
γ = the density of water,
g = the force of gravity,
ζ = a coefficient depending upon the form of the body, and to be determined by experiment.

If we take for units the foot and the pound, we have

$$g = 32.166 \text{ feet},$$
$$\gamma = 62.5 \text{ pounds}.$$

At the place where the western pier will stand, we found the bottom as follows:

April 2, 1867,	stage of water	18.6 ft.	below	Directrix,	bottom	50 ft.	below	Directrix.
June 2, 1867,	" "	7.6 ft.	"	"		68 ft.	"	"
Nov. 28, 1867,	" "	28.4 ft.	"	"		50 ft.	"	"

On the 2d June, when the water was 7.6 feet below Directrix, or 11 ft. above the stage of 2d April, the bottom had been washed out 18 ft. deeper. But the water may again rise 15.18 ft. higher, and equal the flood of 1844, which was 7.58 ft. above the Directrix. It will therefore be no unreasonable supposition to assume the greatest possible scouring in the open river as 50 feet below the ordinary bottom, and if the river is confined between abutments, and obstructed by piers, it will not be too much to assume a possible scouring to a depth of 60 feet below the ordinary bottom. Accordingly, we can get a maximum depth of water for our piers of nearly 120 feet. For the calculation, we shall assume the highest stage of water at spring-line (8.5 ft. above Directrix) and the bottom of the river at 120 ft. below spring-line. For simplicity, we shall also neglect the offset at the line GH, Fig. 16, and suppose the batter (= 0.065) to extend unbroken to the bottom.

The normal sectional area, F, of the obstruction is then, in round numbers,

$$F = \tfrac{1}{2}(49 + 34)\,120 = 4980 \text{ square feet}.$$

The greatest velocity of the current, at the highest stage of water is 12.5 ft. per second, measured 3 ft. below the surface. With a greater

depth, the velocity decreases, but in the upper regions very slowly, and for safety we will take throughout

$$v = 12.5.$$

For a prism of the length l in the direction of the current, and the section F normal to it, Weisbach gives

$$\text{for } \frac{l}{\sqrt{F}} = 0, \quad 1, \quad 2, \quad 3, \qquad \zeta = 1.86 \quad 1.47 \quad 1.35 \quad 1\,33$$

according to experiments of Buat and Duchemin,

and according to Borda, the impact on a cylinder perpendicular to its axis is about one half of the impact on a parallellopipedon of the same dimensions.

As the horizontal section of our piers has semicircular ends, we can apply to it without much mistake the rule for a cylinder.

The average length of piers (in the direction of the current) is about 90 feet, and we have

$$\frac{l}{\sqrt{F}} = \frac{90}{\sqrt{4980}} = \frac{90}{70.6} = 1.3.$$

Our ζ would therefore be between $\frac{1}{2} \times 1.47$, and $\frac{1}{2} \times 1.35$, and we will assume

$$\zeta = 0.71,$$

with which we find

$$P = 0.71 \times \frac{(12.5)^2}{2 \times 32.166} \times 4980 \times 62.5 \text{ pounds},$$

$$= 536750 \text{ pounds}, = 268 \text{ tons}.$$

Considering the increase of width of pier with greater depth about equivalent to the decrease of velocity of current, we may assume the resultant P acting at a distance of 60 ft. below the spring-line.

There is no danger of sliding by the impact of the stream, for the weight of masonry above spring-line alone amounts to more than 8000 tons.

The pressure of a tornado, tearing up trees, etc., can be assumed to be 50 pounds per square foot. Such a tornado coming downwards at the highest stage of water will act at the pier on an area of

$$[11.7 + 15.83]\, 63.5 = 1748 \text{ square feet},$$

and its force on the pier will be

$$1748 \,.\, 50 = 87400 \text{ pounds} = 43.7 \text{ tons}.$$

The action of wind on the superstructure can take place on 40 square feet per running foot, which gives a force of 1 ton per running foot. On one pier comes the pressure for one span of 515 feet, or 515

tons. This force is exerted partly at the top of the pier, partly at the height of the lower roadway, and partly at the spring of the arch. We will assume the resultant at 30′ above the spring-line. The resultant of the horizontal pressure of the wind against the pier can also nearly enough be assumed at 30′ above the spring-line, and the whole effect of wind on the pier can be represented as a horizontal force of

$$515 + 44 = 559 \text{ tons},$$

at 30′ distance above spring-line.

The quantity of masonry in one pier above CD is in round numbers,

$$= 110000 \text{ cub. feet.}$$

and its weight will be $\frac{110000 \times 150}{2000} = 8250$ tons.

The quantity of masonry between CD and the bottom is

$$= 368000 \text{ cub. feet,}$$

and its weight will be $\frac{368000 \times 150}{2000} = 27600$ tons.

Total weight of pier above the bottom = 35850 tons.

The weight of superstructure resting on one pier (259 tons per rib) is $8 \times 259 = 2072$ tons.

The total vertical force on the piece above the bottom is

$$V = 35850 + 2075 = 37925 \text{ tons,}$$

acting in the centre line of the pier.

The horizontal force is

$$H = 268 \text{ (impact of stream)} + 559 \text{ (force of wind)} = 793 \text{ tons.}$$

The distance, x, of the point where the resultant, R, of V and H meets the bottom-plane, from the centre of the pier, is determined by the equation

$$x = \frac{1}{V}[268 \times 60 + 559 \times 150] = \frac{16080 + 83850}{37925} = 2.64. \text{ ft.}$$

The small amount of x in comparison with the length of the pier, shows that the pier could stand much greater horizontal forces without the least danger.

Effect of Ice.

We have to distinguish between two cases,

1. A heavy mass of ice comes down the river with a certain velocity and acts on the pier with an impact.

2. A gorge is formed above the piers and supported by them, and, in consequence of it, the river above rises, and acts on the gorge, and thus on the piers, with a hydrostatic pressure.

1. IMPACT OF ICE.

The ice often moves off in fields as large as 500 by 1000 feet in area, and 12 to 14 inches thick. The velocity of such great masses of ice probably never exceeds 3 feet per second.

We shall assume that a field of ice, of 500000 square feet area and 1.2 feet thick, comes with its centre against a pier with a velocity of 3 feet per second, and that the pier resists with its whole width (which will occur if the ice has an excavation fitting to the pier, or if the first ice has been crushed). We now put

w = the weight of one cubic foot of ice in pounds = 55,
W = the weight of the whole field of ice = 33000000 pounds,
g = the force of gravity = 32.166 feet,
b = the width of pier at the height of the ice = 38 feet (the ice generally comes no higher than 24 feet below Directrix),
δ = the thickness of the ice = 1.2 feet,
P = the horizontal resistance exerted by the pier,
s = the linear motion of the ice during a time, t, after the first contact,
v = the velocity of the ice at the end of the time t.

The resistance of the pier necessary to bring the ice to rest instantaneously would far surpass the crushing strain of ice; therefore the ice at the pier will be crushed, and the pier will cut for some distance into the ice. The work done by P, during an infinitely small time, dt, in which W moves through a distance, ds, and the velocity of W is brought to $v + dv$, is Pds. By this the velocity, v, of a mass, $\frac{w}{g} b \delta ds$, was wholly destroyed, and the velocity, v, of the mass, $\left[\frac{W}{g} - \frac{w}{g} b \delta s\right]$, has become $v + dv$. We have therefore the equation,

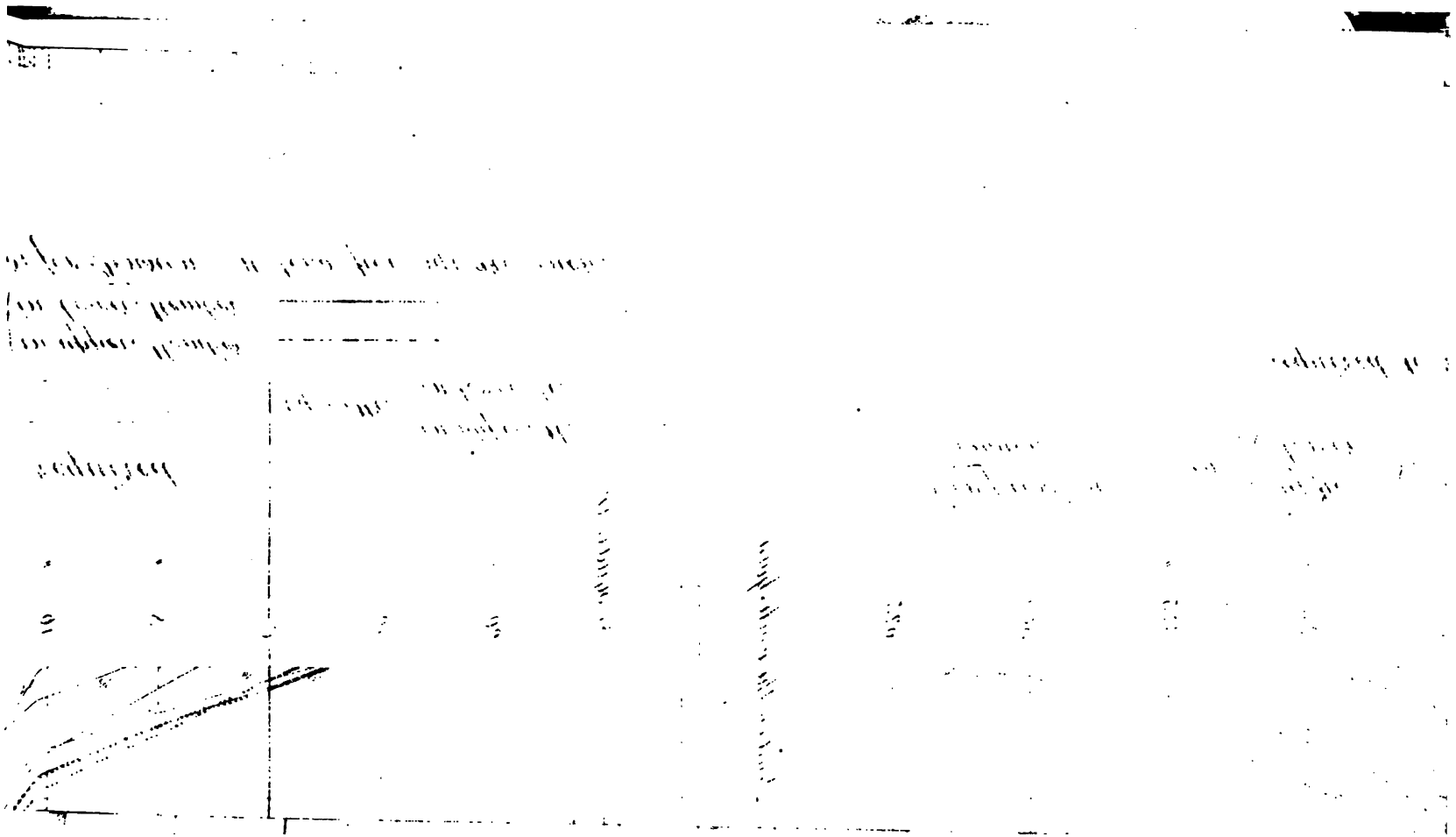
$$Pds = -\left[\frac{W}{g} - \frac{w}{g} b \delta s\right] v\, dv + b\delta . \frac{w}{g} . \frac{v^2}{2} ds;$$

from which, since

$$\frac{v\, dv}{ds} = \frac{ds}{dt} . \frac{dv}{ds} = \frac{dv}{dt},$$

we deduce,

$$P = -\left[\frac{W}{g} - \frac{w}{g} b \delta s\right] \frac{dv}{dt} + b\delta \frac{w}{2g} . v^2,$$



or,

$$\frac{dv}{dt} = -\frac{P - \frac{w}{2g} \cdot b\,\delta\,v^2}{\frac{W}{g} - \frac{w}{g} b\,\delta\,s}.$$

If k is the crushing strain of ice per square foot, P will be always greater or equal to $b\,\delta\,.\,k$, and we can put

$$P = b\,\delta\,[k + p],$$

where p depends upon the velocity v, and is derived from the circumstance that the particles of ice which come in contact with the pier must not only be crushed, but also brought to rest immediately. We have, therefore, the equation,

$$b\,\delta\,p\,ds = b\,\delta\,ds\,.\,\frac{w}{2g}v^2, \quad \text{or,} \quad p = \frac{w}{2g}v^2;$$

and hence,

$$P = b\,\delta\left(k + \frac{w}{2g}v^2\right),$$

$$\frac{dv}{dt} = \frac{b\,\delta\,k}{\frac{W}{g} - \frac{w}{g} b\,\delta\,s}.$$

If we assume the crushing strain of ice to be as great as that of brick, = 600 pounds per square inch, we have

$$k = 600 \times 144 = 86400 \text{ pounds per square foot;}$$

and for $v = 3$ feet, the greatest value of P,

$$\text{(max.) } P = 38 \times 1.2\left[86400 + \frac{55}{64.332} \times 9\right] = 3939840 + 351 = 3940191 \text{ lbs.}$$
$$= 1970 \text{ tons.}$$

Further,

$$\frac{dv}{dt} = -\frac{38 \times 1.2 \times 86400}{\frac{33000000}{32.166} - \frac{55}{32.166} \times 38 \times 1.2 \times s} = -\frac{3939840}{1025928 - 77.968\,.\,s};$$

or very nearly, $\frac{dv}{dt} = -\frac{3939840}{1025928}^{*} = -3.84.$

By integration we get

$$v = \frac{ds}{dt} = -3{,}84\,t + v_0,$$
$$s = v_0\,t - \tfrac{1}{2} \times 3.84\,t^2;$$

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* Exact for $s = 0$, or the beginning of the impact.

and if we call

t_1 the time in which the ice is brought to rest, and

s_1 the distance which the pier enters into the ice, we get

$$t_1 = \frac{3}{3.84} = 0.78 \text{ seconds};$$

$$s_1 = 3 \times \frac{3}{3.84} - \frac{1}{2} \times 3.84 \times \frac{3^2}{3.84^2} = \frac{1}{2} \times 3 \times 0.78$$

$$= 1.17 \text{ feet.}$$

The horizontal force P, which the pier has to stand by the impact of ice, is so small in comparison with the effect of load, that its stability is established without further calculations.

2. EFFECT OF A GORGE OF ICE.

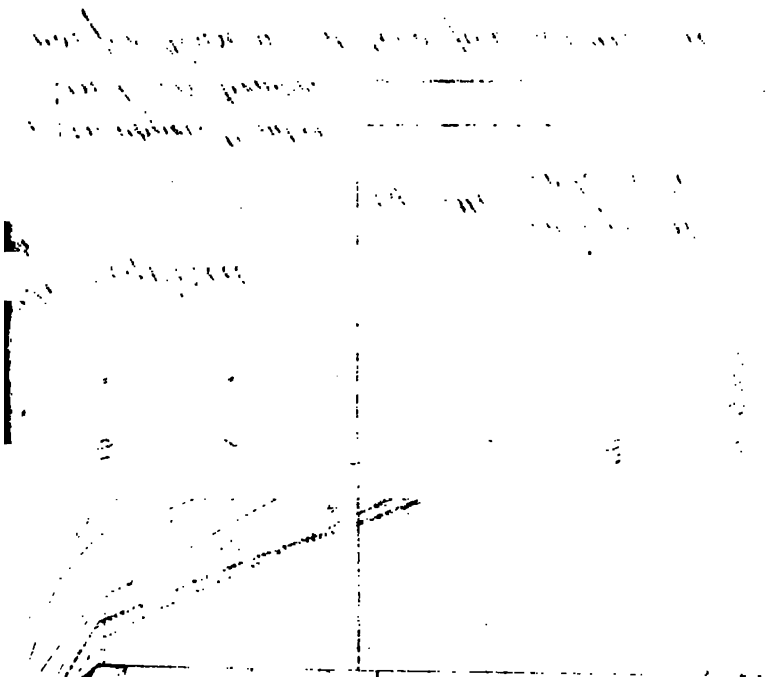
If we assume that the ice may at some time be gorged to such an extent as to cause the sand to be cut away to the rock between the piers, the pressure of this gorged ice against the piers, and its tendency to move them down stream or overturn them, will be only equivalent to the hydrostatic pressure due to the difference between the level of the water immediately above and below the bridge. If it were possible for this difference to equal 10 feet, the pressure exerted would then be

$$10 \times \frac{10 \times 62.5}{2} = 3125 \text{ pounds per linear foot.}$$

If the gorge is resting against the piers, one pier has to support a length of $515_{(\text{span})} + 37.5_{(\text{pier})} = 552.5$ feet, or the pier has to stand a horizontal force of

$$3125 \times 552.5 = 1726562.5 \text{ pounds,}$$
$$= 863 \text{ tons.}$$

As the weight of masonry above spring-line alone amounts to more than 8000 tons, this force will not be dangerous to the stability of the pier.



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